MATH 247 Calculus 3, Exercises for Chapter 6

1: (a) A function $f(x, y)$ is called **harmonic** if it is a solution to **Laplace's equation**, which is the partial differential equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$. Determine which of the following two functions are harmonic.

(i) $f(x, y) = \ln \sqrt{x^2 + y^2}$ (ii) $f(x, y) = \tan^{-1} \frac{y}{x}$.

(b) Find the Taylor polynomial of degree 2, centred at $(-2, 1)$, for $f(x, y) = (2 - x)\, e^{x+2y}$.

2: (a) Let $z = f(x, y) = x^2 y + 2x^2 + y^2$. Find and classify all the critical points of $f(x, y)$, then find the maximum and minimum values of $z = f(x, y)$ in $D = \{(x, y) | x^2 + y^2 \leq 8\}$.

(b) Find the maximum possible area for a quadrilateral with vertices at $(0, 0)$, $(1 - r, 0)$, $(1 - r + r\cos\theta, r\sin\theta)$ and $(0, r\sin\theta)$, with $0 \leq r \leq 1$ and $0 \leq \theta \leq \frac{\pi}{2}$.

3: Let $u = f(x, y, z) = x^2 + xy + y^2 + 3yz^2 + 6z^2$. Find and classify all the critical points of $f(x, y, z)$, then find the maximum and minimum values of u with $-1 \leq x \leq 3$, $-4 \leq y \leq 0$ and $z = 1$.