MATH 247 Calculus 3, Exercises for Chapter 7

1: (a) Find $\displaystyle\int_D xy\ dA$ where D is the region bounded by $x + y = -1$ and $x + y^2 = 1$.

(b) Find $\displaystyle\int_D x^2 + y\ dV$ where D is the tetrahedron bounded by $x + y + z = 2$, $z = 2$, $x = 1$ and $y = x$.

2: (a) Find $\displaystyle\int_D e^{x-y}\ dA$ where D is the parallelogram with vertices at $(1, 1)$, $(3, 2)$, $(4, 5)$ and $(2, 4)$.

(b) Find $\displaystyle\int_D \sqrt{y^3 + z}\ dV$ where $D = \big\{(x, y, z)\ \big|\ 0 \le x \le 1, \sqrt{x} \le y \le 1, y^3 \le z \le 1\big\}$.

3: (a) Find the volume of the region which lies under the surface $z = e^{x+y}$ and above the triangle in the xy-plane with vertices at $(0, 0)$, $(1, 1)$ and $(0, 2)$.

(b) Find the volume of the region which lies outside the cylinder $x^2 + y^2 = 1$, inside the cylinder $x^2 + y^2 = 2x$ and inside the sphere $x^2 + y^2 + z^2 = 4$.

4: (a) Find the mass of the solid tetrahedron with vertices at $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ with density given by $f(x, y, z) = 1/(1 + x)$.

(b) Find the mass of the solid which lies inside the cone $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 4$ with density given by $f(x, y, z) = 2 - z$.

5: (a) A cord, carrying an unevenly distributed charge, is wound around the cone $z = \sqrt{x^2 + y^2}$ following the curve $(x, y, z) = \alpha(t) = (t\cos t, t\sin t, t)$ with $0 \le t \le 4$. The charge density (charge per unit length) of the cord at position (x, y, z) is given by $f(x, y, z) = z$. Find the total charge of the cord.

(b) The surface obtained by revolving the circle $(x - 1)^2 + z^2 = 1$ in the xz-plane about the z-axis can be given parametrically by

$$(x, y, z) = \sigma(\theta, \phi) = \Big((1 + \cos\phi)\cos\theta\,,\ (1 + \cos\phi)\sin\theta\,,\ \sin\phi\Big).$$

with $0 \le \theta \le 2\pi$ and $0 \le \phi \le 2\pi$. Find the mass of this surface given that its density (mass per unit area) at position (x, y, z) is given by $f(x, y, z) = 1 + z^2$.