1: Let $f:\mathbb{R}\to\mathbb{R}$. Suppose that $f(x+y)=f(x)+f(y)$ for all $x,y\in\mathbb{R}$.

(a) Show that $f(rx)=rf(x)$ for all $r\in\mathbb{Q}$.

(b) Show that f is continuous at 0 if and only if f is continuous on $\mathbb{R}$.

(c) Show that if f is continuous at 0 then there exists $m\in\mathbb{R}$ such that $f(x)=mx$ for all $x\in\mathbb{R}$.

2: (a) Define $f,g:\mathbb{R}\to\mathbb{R}$ by $f(x)=x^3$ and $g(x)=\sqrt[3]{x}$. Show that g is uniformly continuous but f is not.

(b) Let $f:(a,b)\subseteq\mathbb{R}\to\mathbb{R}$. Show that f is uniformly continuous on (a,b) if and only if there exists a continuous function $g:[a,b]\subseteq\mathbb{R}\to\mathbb{R}$ such that $g(x)=f(x)$ for all $x\in(a,b)$.

3: (a) Let $f,g:[a,b]\subseteq\mathbb{R}\to\mathbb{R}$ be integrable and let $c\in\mathbb{R}$. Prove, from Definition 3.3, that the functions $f+g$ and cf are inegrable on $[a,b]$ with $\int_a^b(f+g)=\int_a^b f+\int_a^b g$ and $\int_a^b cf=c\int_a^b f$.

(b) Let $f:[a,b]\subseteq\mathbb{R}\to\mathbb{R}$ be any function (not necessarily bounded). Show that if f is integrable (according to Definition 3.3, but without the assumption that f is bounded) then f must be bounded.

4: Determine (with proof) which of the following statements are true for all bounded functions $f:[0,1]\to\mathbb{R}$.

(a) If $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{1}{n}f\left(\frac{k}{n}\right)$ exists then f is integrable on $[0,1]$.

(b) For all $a,b\in[0,1]$ and $S\in\mathbb{R}$, if $f(a)<S<f(b)$ then there exists a partition $X=\{x_0,x_1,\cdots,x_n\}$ of $[0,1]$ and there exist sample points $t_k\in[x_{k-1},x_k]$ such that $\sum\limits_{k=1}^{n}f(t_k)\Delta_k x=S$.