PMATH 336 Intro to Group Theory, Assignment 1 Due Tues May 21, 11:00 pm

1: Determine which of the following are groups and which of the groups are abelian.

(a) $G = \{1, 4, 7, 10, 13\}$ under multiplication modulo 15.

(b) $G = \{p(x) = ax + b \big| a \in U_4, b \in \mathbb{Z}_4\}$ under composition of polynomials.

(c) $G = \{x \in \mathbb{R} \big| x > 1\}$ under the operation $*$ given by $x * y = xy - x - y + 2$.

2: Let G be a group with identity e.

(a) Let $a, b \in G$ with $a^4 = e$ and $ab = ba^2$. Show that $a = e$.

(b) Let $a, b \in G$ with $a^{16} = b^9$ and $a^{25} = b^{14}$. Show that $a = b$.

(c) Let $a, b \in G$ with $|a| = 2$, $b \neq e$ and $ab = b^2 a$. Find $|b|$ and $|ab|$.

3: (a) Write out the multiplication table for U_{20}.

(b) Find the order of each element in U_{20}.

(c) Solve $x^3 y^6 = 3$ for $x, y \in U_{20}$.

4: When R is a commutative ring (with identity), the set $M_n(R)$ of $n \times n$ matrices with entries in R is a ring (with identity) under matrix addition and matrix multiplication. The subsets $GL_n(R) = \{A \in M_n(R) \mid \det A \in R^*\}$, $SL_n(R) = \{A \in M_n(R) \mid \det A = 1\}$, $O_n(R) = \{A \in M_n(R) \mid A^T A = I\}$ and $SO_n(R) = \{A \in O_n(R) \mid \det A = 1\}$ are groups (with identity) under matrix multiplication.

(a) Find $\big|SL_2(\mathbb{Z}_5)\big|$.

(b) Find every element of order 2 in $SL_2(\mathbb{Z}_5)$.

(c) Find $\big|O_2(\mathbb{Z}_5)\big|$ and $\big|SO_2(\mathbb{Z}_5)\big|$.