PMATH 336 Intro to Group Theory, Assignment 2 — Due Tues June 4, 11:00 pm

1: Sketch a picture of each of the following subsets of $\mathbb{C}^*$ and, in parts (c) and (d), determine whether the given subset is a subgroup (under multiplication).

(a) $\left\langle \frac{i-1}{\sqrt{2}} \right\rangle$

(b) $\langle 1+i \rangle$

(c) $\left\{ z \in \mathbb{C}^* \,\middle|\, z^8 = |z|^8 \right\}$ (where $|z|$ denotes the usual norm of z)

(d) $\left\{ re^{i\theta} \in \mathbb{C}^* \,\middle|\, r > 0\,,\ \theta = \frac{\pi}{2} \log_2 r \right\}$.

2: Consider the group $D_6 = \left\{ I, R_1, R_2, R_3, R_4, R_5, F_0, F_1, F_2, F_3, F_4, F_5 \right\}$.

(a) Make the multiplication table for D_6.

(b) Find the order of each element in D_6.

(c) Solve the equation $X^2 Y^3 = R_1$ for X and Y in D_6.

3: (a) Show that U_{25} is cyclic.

(b) List all the elements and all the generators of every subgroup of U_{25}.

(c) Find a non-cyclic subgroup of order 4 in U_{20}.

4: Let G be a multiplicative group and let $a \in G$ with $|a| = 1400$.

(a) Determine the number of subgroups of $\langle a \rangle$.

(b) Determine the number of elements $x \in \langle a \rangle$ with $|x| \leq 10$.

(c) List all the elements $x = a^k \in \langle a \rangle$ with $x^{52} = 1$.

(d) Find the number of pairs (x, y) with $x, y \in \langle a \rangle$ such that $x^{10} = y^{35}$ in $\langle a \rangle$.