PMATH 336 Intro to Group Theory, Assignment 3 Due Tues June 18, 11:00 pm

**1:** Let $\alpha, \beta \in S_8$ be given by the following table of values:

| $k$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $\alpha(k)$ | 4 | 6 | 3 | 5 | 7 | 8 | 1 | 2 |
| $\beta(k)$ | 2 | 6 | 7 | 4 | 8 | 3 | 1 | 5 |

For each of the following permutations, write the permutation as a product of disjoint cycles and determine its order and its parity.

(a) $\alpha$ (b) $\beta$ (c) $\alpha\beta$ (d) $\left(\alpha\beta^{-1}\right)^{20}$

**2:** (a) Find the maximum of the orders of the elements in $S_8$.

(b) Find the number of elements of order 6 in $S_8$.

(c) Find the number of cyclic subgroups of order 6 in $A_8$.

**3:** Let $\alpha = (1234)(5678)$ and $\beta = (123)(456)$ in $S_8$.

(a) Express $\alpha$ as a product of 2-cycles and as a product of 3-cycles.

(b) Find $|Cl(\beta)|$, that is find the number of elements in the conjugacy class of $\beta$.

(c) Find all the elements $\sigma \in S_8$ such that $\sigma^2 = \beta$.

**4:** (a) Find the number of elements of each order in $A_4 \times D_4$.

(b) Find the number of elements of each order in $\mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_6$.