PMATH 336 Intro to Group Theory, Assignment 4 Due Tues July 2, 11:00 pm

1: (a) Let $G = \left\{ \left(\begin{smallmatrix} a & -b \\ b & a \end{smallmatrix} \right) \in GL_2(\mathbb{R}) \middle| \, a, b \in \mathbb{R} \right\}$. Show that G is a group and that $G \cong \mathbb{C}^*$.

(b) Suppose that $\phi : U_{30} \to \mathbb{Z}_2 \times \mathbb{Z}_4$ is a group isomorphism. Find $\phi(19)$.

2: Show that no two of the groups $\mathbb{Z}_{12}$, U_{28}, D_6 and A_4 are isomorphic.

3: (a) The elements of D_4 permute the 4^{th} roots of unity, namely $e^{i\,k\pi/2}$ for $k \in \mathbb{Z}_4 = \{1, 2, 3, 4\}$, and so each element of D_4 determines a permutation of $\mathbb{Z}_4$. In this way, we can think of D_4 as a subgroup of S_4. To be precise, there is an isomorphism $\phi : D_4 \to H = \phi(D_4) \leq S_4$ which is determined by $\phi(R_1) = (1234)$ and $\phi(F_0) = (13)$. Find all the elements of $H = \phi(D_4)$.

(b) Let $f : U_9 \to \{1, 2, 3, 4, 5, 6\}$ be the bijection given by

x	1	2	4	5	7	8
$f(x)$	1	2	3	4	5	6

and define $\phi : U_9 \to S_6$ by $\phi(a) = f \circ L_a \circ f^{-1}$, that is $\phi(a)(k) = f\big(a\, f^{-1}(k)\big)$. List all the elements in $\phi\big(U_9\big)$.

4: (a) Find $\text{Inn}(D_4)$ (that is, list all the distinct elements in $\text{Inn}(D_4)$).

(b) Show that $\big|\text{Aut}(D_4)\big| > \big|\text{Inn}(D_4)\big|$.