PMATH 336 Introduction to Group Theory, Exercises for Chapter 1

1: (a) Write down the multiplication table for U_{18}.

(b) Find the order of each element in U_{18}.

2: Determine which of the following are groups.

(a) $X = \{(x, y) \in \mathbb{R}^2 | x^2 = y^2\}$ under vector adition.

(b) $G = \{1, 3, 5, 7, 9\}$ under multiplication modulo 10.

(c) $\mathbb{R}$ under the operation $*$ given by $x * y = x + y + 1$.

(d) $H = \{1, 2, 4, 8, 16\}$ under the operation $*$ given by $a * b = \gcd(a, b)$.

3: Let G be a group with identity e. Prove each of the following statements.

(a) If $(\forall a, b, c \in G \quad ab = ca \Rightarrow b = c)$ then G is abelian.

(b) If $(\forall a, b \in G \quad (ab)^2 = a^2 b^2)$ then G is abelian.

(c) If $(\forall a \in G \quad a^2 = e)$ then G is abelian.