PMATH 336 Introduction to Group Theory, Exercises for Chapter 2

1: In D_n, for $k \in \mathbb{Z}_n$, we write R_k for the rotation in about the point $(0,0)$ by the angle $\frac{2\pi k}{n}$, and we write F_k for the reflection in the line through $(0,0)$ and $\left(\cos\frac{\pi k}{n}, \sin\frac{\pi k}{n}\right)$.

(a) Find all values of $k \in \mathbb{Z}_6$ such that $F_3 R_k F_1 = R_k$ in D_6.

(b) Find the centralizer of F_1 in D_6.

2: (a) Find $\left|GL(3, \mathbb{Z}_2)\right|$

(b) List all the elements in $SO(3, \mathbb{Z}_2)$.

3: (a) Show that U_{26} is cyclic.

(b) List all the elements and all the generators in every subgroup of U_{26}.

4: (a) Determine the number of subgroups of $\mathbb{Z}_{12,000}$.

(b) Find the number of elements of even order in $\mathbb{Z}_{12,000}$.

5: (a) Find the number of elements of each order in $\mathbb{Z}_3 \times \mathbb{Z}_6$.

(b) List all the elements in every cyclic subgroup of $\mathbb{Z}_3 \times \mathbb{Z}_6$.

(c) List all the elements in every non-cyclic subgroup of $\mathbb{Z}_3 \times \mathbb{Z}_6$. Explain why your list is complete.