PMATH 336 Introduction to Group Theory, Exercises for Chapter 4

1: (a) Define $\phi : \mathbb{Z}_{60} \to U_{45}$ by $\phi(k) = 2^k$. Show that ϕ is a group homomorphism, and find $\mathrm{Ker}(\phi)$ and $\mathrm{Im}(\phi)$.

(b) Define $\psi : SL(n,\mathbb{R}) \times \mathbb{R}^* \to GL(n,\mathbb{R})$ by $\psi(A,t) = tA$. Show that ψ is a group homomorphism and find $\mathrm{Ker}(\psi)$ and $\mathrm{Im}(\psi)$.

2: Show that no two of the groups $\mathbb{Z}_8$, U_{16}, D_4 and ${\mathbb{Z}_2}^3$ are isomorphic.

3: Find the number of elements of each order in $U_{55} \times A_4$.

4: (a) Find the number of homomorphisms from $\mathbb{Z}_{12}$ to D_9.

(b) Find the number of homomorphisms from D_9 to $\mathbb{Z}_{12}$.

5: (a) Find the number of homomorphisms from $\mathbb{Z}_4 \times \mathbb{Z}_6$ to itself.

(b) Find the number of homomorphisms from $\mathbb{Z}_4 \times \mathbb{Z}_6$ to D_{12}.

6: Let $f : S_3 \to \{1,2,3,4,5,6\}$ be the bijection given by the table of values

α	(1)	(12)	(13)	(23)	(123)	(132)
$f(\alpha)$	1	2	3	4	5	6

and let $\phi : S_3 \to S_6$ be the isomorphism given by $\phi(\alpha) = f \circ L_\alpha \circ f^{-1}$, where $L_\alpha(\beta) = \alpha\beta$ for all $\beta \in S_3$. List all the elements in $\phi(S_3)$

7: Find $\big|\mathrm{Inn}(Q)\big|$, where $Q = \{1, i, j, k, -1, -i, -j, -k\}$ is the **quaternionic** group, which has the following multiplication table $\big(Q$ is not isomorphic to any group from Exercise 2$\big)$.

	1	i	j	k	-1	$-i$	$-j$	$-k$
1	1	i	j	k	-1	$-i$	$-j$	$-k$
i	i	-1	k	$-j$	$-i$	1	$-k$	j
j	j	$-k$	-1	i	$-j$	k	1	$-i$
k	k	j	$-i$	-1	$-k$	$-j$	i	1
-1	-1	$-i$	$-j$	$-k$	1	i	j	k
$-i$	$-i$	1	$-k$	j	i	-1	k	$-j$
$-j$	$-j$	k	1	$-i$	j	$-k$	-1	i
$-k$	$-k$	$-j$	i	1	k	j	$-i$	-1