PMATH 336 Introduction to Group Theory, Exercises for Chapter 5

1: (a) List all the elements in every coset of $H = \{1, 9, 17, 33\}$ in $G = U_{40}$.

(b) List all the elements in every left coset and every right coset of $H = \langle(1234)\rangle$ in $G = S_4$.

2: Find four distinct subgroups $G_i \leq S_5$, with $i = 1, 2, 3, 4$, such that $\mathrm{orb}_{G_i}(1) = \{1, 2, 4\}$. For each of the four subgroups G_i, find $\mathrm{stab}_{G_i}(1)$.

3: Let $G = \mathbb{Z}^2$ and let $H = \big\langle(3, 2), (6, 8)\big\rangle = \big\{k(3, 2) + l(6, 8)\big|k, l \in \mathbb{Z}\big\}$. For each pair $(a, b) \in G$ with $0 \leq a < 6$ and $0 \leq b < 2$, find the order of $(a, b) + H$ in the group G/H.

4: Determine which of the following five subgroups of S_4 are normal: $\big\langle(12)\big\rangle$, $\big\langle(12)(34)\big\rangle$, $\big\langle(123)\big\rangle$, $\big\langle(1234)\big\rangle$ and $\mathrm{stab}_{S_4}(1)$.