

PMATH 336 Introduction to Group Theory, Exercises for Chapter 7

- 1:** Let $a = (2, 6)$, $b = (3, -1)$ and $c = (4, 2)$. Find the image of triangle with vertices at a , b and c under the isometry $\mathbf{R}_{(4,6), \frac{\pi}{2}} \mathbf{G}_{(4,2), x-2y+5=0}$.
- 2:** Express the composite $\mathbf{R}_{(1,4), \frac{\pi}{2}} \mathbf{F}_{x+3y=3}$ as a single glide-reflection.
- 3:** Find the symmetry group of each of the following subsets of \mathbb{R}^2 .
 - (a) $X = \{(1, 1), (5, 3)\}$.
 - (b) $Y = L \cup M$ where L is the line $x + y = 1$ and M is the line $x + y = 3$.
- 4:** Let X be the polyhedron whose 12 vertices are at $(\pm 2, 0, \pm 2)$ and $(\pm 1, \pm\sqrt{3}, \pm 2)$ (X is a prism whose two ends are regular hexagons). Determine whether the rotation group of X is isomorphic to \mathbb{Z}_n , D_n , A_4 , S_4 or to A_5 .
- 5:** (a) How many 8-bead necklaces can be made (up to D_8 symmetry) using beads of 2 colours.
(b) How many ways (up to rotational symmetry) can the faces of a regular octahedron be coloured using 2 colours?