PMATH 347 Groups and Rings, Assignment 2 Due Tues June 4, 11:00 pm

1: (a) Find $Z(D_n)$.

(b) Find $Z\big(GL_n(\mathbb{R})\big)$.

(c) Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \in GL_3(\mathbb{Z}_5)$. Find the order of the centralizer of A in $GL_3(\mathbb{Z}_5)$.

2: (a) Show that U_{22} is cyclic, U_{15} is not cyclic, and U_{2^n} is not cyclic for $n \geq 3$.

(b) Find the number of cyclic subgroups of $\mathbb{Z}_9 \times \mathbb{Z}_{15}$.

(c) Find a non-cyclic proper subgroup of $\mathbb{Z}_9 \times \mathbb{Z}_{15}$.

3: (a) Let G be a group and let $a, b \in G$. Show that $\langle ab, a^2b \rangle = \langle a, b \rangle$.

(b) Let $a, b \in \mathbb{Z}$ and let $d = \gcd(a, b)$. Show that in the group $\mathbb{Z}$ we have $\langle a, b \rangle = \langle d \rangle$.

(c) Show that every finitely generated subgroup of $\mathbb{Q}$ is cyclic.

(d) Find a non-cyclic proper subgroup of $\mathbb{Q}$.

4: (a) List all of the elements $X \in D_{28}$ such that $F_5X^3 = X^9F_{13}$.

(b) Find all subgroups of D_n.