PMATH 347 Groups and Rings, Assignment 4 Due Tues July 2, 11:00 pm

1: Show that no two of the groups $\mathbb{Z}_{24}$, U_{39}, D_{12}, S_4 and $SL(2,\mathbb{Z}_3)$ and are isomorphic.

2: (a) Show that no two of the groups $\mathbb{Q}$, $\mathbb{Q}^*$ and $\mathbb{Q}^+$ are isomorphic.

(b) Determine whether, for all $n,m\in\mathbb{Z}^+$, the groups $\mathbb{Q}^n$ and $\mathbb{Q}^m$ are isomorphic if and only if $n=m$.

(c) Determine whether any two of the rings $\mathbb{Q}^2$, $\mathbb{Q}[\sqrt{2}\,]$ and $\mathbb{Q}[\sqrt{3}\,]$ are isomorphic (as rings).

3: (a) Find a subgroup of S_4 which is isomorphic to U_8.

(b) Find a subgroup of S_4 which is isomorphic to $\mathrm{Aut}\big(U_8\big)$.

(c) Show that $\mathrm{Aut}(\mathbb{Z}_n)\cong U_n$.

4: (a) Find a formula for the number of group homomorphisms $\phi:\mathbb{Z}_n\to\mathbb{Z}_m$, where $n,m\in\mathbb{Z}^+$.

(b) Find a formula for the number of group homomorphisms $\phi:\mathbb{Z}_n\times\mathbb{Z}_m\to\mathbb{Z}_\ell$, where $n,m,\ell\in\mathbb{Z}^+$.

(c) For a positive integer n, let $\omega(n)$ denote the number of distinct prime factors of n. Find a formula (in terms of n and m, using ω) for the number of ring homomorphisms $\phi:\mathbb{Z}_n\to\mathbb{Z}_m$, where $n,m\in\mathbb{Z}^+$.