

PMATH 347 Groups and Rings, Exercises for Chapter 1

- 1:** (a) Write down the multiplication table for U_{18} .
(b) Find the order of each element in U_{18} .
- 2:** Determine which of the following are groups.
- (a) $X = \{(x, y) \in \mathbb{R}^2 \mid x^2 = y^2\}$ under vector addition.
 - (b) $G = \{1, 3, 5, 7, 9\}$ under multiplication modulo 10.
 - (c) \mathbb{R} under the operation $*$ given by $x * y = x + y + 1$.
 - (d) $H = \{1, 2, 4, 8, 16\}$ under the operation $*$ given by $a * b = \gcd(a, b)$.
- 3:** Let G be a group with identity e . Prove each of the following statements.
- (a) If $(\forall a, b, c \in G \quad ab = ca \Rightarrow b = c)$ then G is abelian.
 - (b) If $(\forall a, b \in G \quad (ab)^2 = a^2b^2)$ then G is abelian.
 - (c) If $(\forall a \in G \quad a^2 = e)$ then G is abelian.