

PMATH 347 Groups and Rings, Exercises for Chapter 11

- 1:** (a) Find all the units in $\mathbb{Z}[\frac{1}{2} + \frac{\sqrt{3}}{2}i]$.
(b) Find 10 units in $\mathbb{Z}[\sqrt{3}]$.
- 2:** Determine which of the following elements are irreducible in $\mathbb{Z}[\sqrt{3}i]$.
(a) $3 + 2\sqrt{3}i$ (b) $2 + 3\sqrt{3}i$ (c) 5 (d) 7
- 3:** (a) Show that $2 + \sqrt{5}i$ is irreducible but not prime in $\mathbb{Z}[\sqrt{5}i]$.
(b) Draw a picture of each of the ideals $\langle 2 \rangle$, $\langle 1 + \sqrt{3}i \rangle$ and $\langle 2, 1 + \sqrt{3}i \rangle$ in $\mathbb{Z}[\sqrt{3}i]$.
- 4:** (a) Determine whether the set $\{ \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} \}$ is linearly independent in $M_2(\mathbb{Z}_5)$.
(b) Find the line of intersection of the planes $x + 3y + z = 1$ and $2x + y + 4z = 1$ in $(\mathbb{Z}_5)^3$.
(c) How many invertible matrices are there in $M_2(\mathbb{Z}_2)$?