

PMATH 347 Groups and Rings, Exercises for Chapter 12

- 1:** (a) Let $f = 5x^4 + 3x^3 + 1$ and $g = 3x^2 + 2x + 1$ in $\mathbb{Z}_7[x]$. Find q and r in $\mathbb{Z}_7[x]$ with $\deg r < \deg g$ such that $f = gq + r$.
- (b) Find a monic polynomial of degree 2 with 4 roots in \mathbb{Z}_{10} .
- 2:** (a) List all the irreducible polynomials of degree less than 4 in $\mathbb{Z}_2[x]$.
- (b) Determine the number of irreducible polynomials of degree 4 in $\mathbb{Z}_2[x]$.
- (c) Determine the number of irreducible polynomials of degree 2 in $\mathbb{Z}_p[x]$ where p is prime.
- 3:** Determine which of the following polynomials are irreducible in $\mathbb{Q}[x]$.
- (a) $x^5 + 9x^4 + 12x^2 + 6$
- (b) $x^4 + x + 1$
- (c) $x^4 + 3x^2 + 3$
- (d) $x^5 + 5x^2 + 1$
- 4:** Factor each of the following polynomials into irreducible factors.
- (a) $f = 4x^4 + x^3 - 3x^2 + 4x - 3 \in \mathbb{Q}[x]$
- (b) $f = x^4 + x^3 + 3x^2 + 2x + 2 \in \mathbb{Q}[x]$
- (c) $f = x^3 + 2x^2 + 2x + 1 \in \mathbb{Z}_7$
- 5:** Find an irreducible polynomial in $\mathbb{Z}[x]$ which is reducible over \mathbb{Z}_2 , \mathbb{Z}_3 , \mathbb{Z}_5 and \mathbb{Z}_7 .