

## PMATH 347 Groups and Rings, Exercises for Chapter 2

- 1:** In  $D_n$ , for  $k \in \mathbb{Z}_n$ , we write  $R_k$  for the rotation in about the point  $(0, 0)$  by the angle  $\frac{2\pi k}{n}$ , and we write  $F_k$  for the reflection in the line through  $(0, 0)$  and  $(\cos \frac{\pi k}{n}, \sin \frac{\pi k}{n})$ .
- (a) Find all values of  $k \in \mathbb{Z}_6$  such that  $F_3 R_k F_1 = R_k$  in  $D_6$ .
  - (b) Find the centralizer of  $F_1$  in  $D_6$ .
- 2:**
- (a) Find  $|GL(3, \mathbb{Z}_2)|$
  - (b) List all the elements in  $SO(3, \mathbb{Z}_2)$ .
- 3:**
- (a) Show that  $U_{26}$  is cyclic.
  - (b) List all the elements and all the generators in every subgroup of  $U_{26}$ .
- 4:**
- (a) Determine the number of subgroups of  $\mathbb{Z}_{12,000}$ .
  - (b) Find the number of elements of even order in  $\mathbb{Z}_{12,000}$ .
- 5:**
- (a) Find the number of elements of each order in  $\mathbb{Z}_3 \times \mathbb{Z}_6$ .
  - (b) List all the elements in every cyclic subgroup of  $\mathbb{Z}_3 \times \mathbb{Z}_6$ .
  - (c) List all the elements in every non-cyclic subgroup of  $\mathbb{Z}_3 \times \mathbb{Z}_6$ . Explain why your list is complete.