

## PMATH 347 Groups and Rings, Exercises for Chapter 3

**1:** In  $S_8$ , let  $\alpha = (1632)(27)(3748)$  and let  $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 5 & 7 & 2 & 8 & 4 & 1 & 6 \end{pmatrix}$ .

(a) Find  $|\alpha|$  and find  $(-1)^\beta$ .

(b) Express each of the permutations  $\alpha^{110}$  and  $\alpha\beta\alpha^{-1}$  as products of disjoint cycles.

**2:** (a) Find the number of elements of each order in  $S_7$  and in  $A_7$ .

(b) Find the number of cyclic subgroups of  $A_7$ .

**3:** Let  $n \geq 3$ .

(a) Show that  $Z(S_n) = \{e\}$ .

(b) Show that every element in  $A_n$  is equal to a product of 3-cycles.