

PMATH 347 Groups and Rings, Exercises for Chapter 5

- 1:** (a) List all the elements in every coset of $H = \{1, 9, 17, 33\}$ in $G = U_{40}$.
(b) List all the elements in every left coset and every right coset of $H = \langle(1234)\rangle$ in $G = S_4$.
- 2:** Find four distinct subgroups $G_i \leq S_5$, with $i = 1, 2, 3, 4$, such that $\text{orb}_{G_i}(1) = \{1, 2, 4\}$. For each of the four subgroups G_i , find $\text{stab}_{G_i}(1)$.
- 3:** Let $G = \mathbb{Z}^2$ and let $H = \langle(3, 2), (6, 8)\rangle = \{k(3, 2) + l(6, 8) \mid k, l \in \mathbb{Z}\}$. For each pair $(a, b) \in G$ with $0 \leq a < 6$ and $0 \leq b < 2$, find the order of $(a, b) + H$ in the group G/H .
- 4:** Determine which of the following five subgroups of S_4 are normal: $\langle(12)\rangle$, $\langle(12)(34)\rangle$, $\langle(123)\rangle$, $\langle(1234)\rangle$ and $\text{stab}_{S_4}(1)$.