SYDE Advanced Math 2, Practice Problem Set 1

1: (a) Verify that $y = x \sin x$ is a solution of the ODE $y\,(y'' + y) = x \sin 2x$.

(b) Find all the solutions of the form $y = ax^2 + bx + c$ to the ODE $\big(y'(x)\big)^2 + 4x = 3y(x) + x^2 + 1$.

2: Consider the IVP $y' = \sin\big(\pi(x+y)\big)$ with $y(-1) = 1$.

(a) Sketch the direction field for the given ODE for $-2 \le x \le 2$ and $-2 \le y \le 2$ and, on the same grid, sketch the solution curves which pass through each of the points $(-1,1)$, $(0,0)$ and $(0,-1)$.

(b) Using a calculator, apply Euler's method with step size $\Delta x = 0.2$ to approximate the value of $f(0)$ where $y = f(x)$ is the solution to the given IVP.

3: Solve each of the following ODEs.

(a) $x\,y' + y = \sqrt{x}$.

(b) $\sqrt{x}\,y' = 1 + y^2$.

(c) $y' = x(y^2 - 1)$.

4: Solve each of the following IVPs.

(a) $x\,y' = y^2 + y$ with $y(1) = 1$.

(b) $x\,y' + 2y = \ln x$ with $y(1) = 0$.

(c) $y' + xy = x^3$ with $y(0) = 1$.

5: Solve each of the following IVPs.

(a) $y' = \dfrac{x+2}{y-1}$ with $y(1) = -2$.

(b) $y' + y \tan x = \sin^2 x$ with $y(0) = 1$.

(c) $y' = \dfrac{y}{x + y^2}$ with $y(3) = 1$.

6: A **Bernoulli** DE is a DE which can be written in the form $y' + py = qy^n$ for some continuous functions p and q and some integer n. The substitution $u = y^{1-n}$ can be used to transform the above Bernoulli DE for $y = y(x)$ into the linear DE $u' + p(1-n)u = q(1-n)$ for $u = u(x)$.

(a) Solve the IVP $y' + y = x\,y^3$, with $y(0) = 2$.

(b) Solve the IVP $xy\,y' + y^2 = 1$ with $y(1) = 2$.

7: A **homogeneous** first order DE is a DE which can be written in the form $y' = F\left(\frac{y}{x}\right)$ for some continuous function F. The substitution $u = \frac{y}{x}$ can be used to transform the above homogeneous DE for $y = y(x)$ into the separable DE $xu' = F(u) - u$ for $u = u(x)$.

(a) Solve the IVP $y' = \dfrac{x^2 + 3y^2}{2xy}$ with $y(1) = 2$.

(b) Solve the IVP $y' = \dfrac{y^2 + 2xy}{x^2}$ with $y(1) = 1$.