# ERRATA: "AN INTRODUCTION TO THE THEORY OF REPRODUCING KERNEL HILBERT SPACES", V. PAULSEN AND M. RAGHUPATHI

## 1. Example 6.14

Example 6.14 is incorrect as stated. The following counterexample is due to H. Woerdeman. We thank J. Mashreghi for bringing our attention to this gap.

Consider the following three matrices:

$$X = \begin{pmatrix} 2/3 & 1/20 \\ 9/16 & 6/10 \end{pmatrix}, Y = \begin{pmatrix} 6/7 & 1/4 \\ 1/60 & 8/10 \end{pmatrix}, Z = \begin{pmatrix} 6/70 & 9/10 \\ 13/14 & 3/100 \end{pmatrix}.$$

Calculation shows that $\|X\|, \|Y\|, \|Z\| < 1$ but that

$$det \begin{pmatrix} (I - X^*X)^{-1} & (I - X^*Y)^{-1} & (I - X^*Z)^{-1} \\ (I - Y^*X)^{-1} & (I - Y^*Y)^{-1} & (I - Y^*Z)^{-1} \\ (I - Z^*X)^{-1} & (I - Z^*Y)^{-1} & (I - Z^*Z)^{-1} \end{pmatrix} = -23322.43 < 0.$$

Curiously, the result is correct for two matrices, which could have lead to the error in the text.

**Proposition 1** (H. Woerdeman)**.** *If $\|X\|, \|Y\| < 1$, then*

$$\begin{pmatrix} (I - X^*X)^{-1} & (I - X^*Y)^{-1} \\ (I - Y^*X)^{-1} & (I - Y^*Y)^{-1} \end{pmatrix},$$

*is positive definite.*

*Proof.* By a Schur complement argument it suffices to show that

$$(I - X^*X)^{-1} - (I - X^*Y)^{-1}(I - Y^*Y)(I - Y^*X)^{-1} > 0,$$

which after multiplying by $(I - X^*Y)$ on the left and by $(I - Y^*X)$ on the right, is equivalent to

$$(I - X^*Y)(I - X^*X)^{-1}(I - Y^*X) - (I - Y^*Y) > 0.$$

Writing $(I - X^*X)^{-1} = \sum_{k=0}^{\infty}(X^*X)^k$, we obtain that

$$(I - X^*Y)(I - X^*X)^{-1}(I - Y^*X) - (I - Y^*Y) =$$

$$(I - X^*Y)(\sum_{k=0}^{\infty}(X^*X)^k)(I - Y^*X) - (I - Y^*Y) =$$

$$(X - Y)^*(X - Y) + X^*(X - Y)(\sum_{k=0}^{\infty}(X^*X)^k)(X - Y)^*X > 0,$$

where the rearrangement of terms is justified by the absolute convergence of the series. □

Institute for Quantum Computing and Department of Pure Mathematics, University of Waterloo, Waterloo, ON, Canada N2L 3G1
*E-mail address*: vpaulsen@uwaterloo.ca