

Suggestions for writing mathematics in scientific papers

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Please send any comments, corrections, or suggestions to me. This document will be updated from time to time. Language evolves. The main content of the current version of the document was prepared in 2022–2023.

The link to the most updated version of this article is

<http://sas.uwaterloo.ca/~wang/files/writing.pdf>

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1 Introduction

The article contains suggestions on academic writing in English for junior academics (especially **PhD students and postdocs**) whose field of study is related to mathematics, finance, economics, operations research, statistics, computer science, or actuarial science. Papers in the above fields are often typeset using \LaTeX . The article has been written mainly for **nonnatives in English**.

I have observed over the years some common mistakes and problems in academic writing, and I have gradually gathered them. Some of these mistakes are obvious to native English speakers but difficult for nonnative speakers. I am not a native speaker, and I have made most of these mistakes myself; this experience helped me to better understand the cause of these innocent mistakes and how to avoid them.

When in doubt about grammar, one should check standard English writing guidelines, for instance, the APA writing style of the American Psychological Association, or another one suitable for the author’s own field. Some suggestions are specific to American English, as British English may have a slightly different convention. However, most of the suggestions, especially on writing mathematics, apply to both.

Most of the points below are meant to be suggestions, and not necessarily strict rules. In what follows, wrong or inappropriate examples are always underlined.

1.1 Acknowledgements

I thank Paul Glasserman, Thomas Mikosch, Vladimir Vovk, Peter Wakker and Ricardas Zitikis for providing very helpful comments and some items in this article. When I started preparing this article, I meant to educate. It turned out that I was educated immensely during the procedure by humans, the Internet, and AI. The one person who had the most impact on my academic writing over many years is Paul Embrechts, to whom I am deeply grateful. All errors and personal preferences are mine.

2 On writing mathematics

2.1 Sentences and mathematical verbs

What I call “mathematical verbs” are logical relations, such as

- “ $=$ ”, which reads as “is equal to” or “equals”;
- “ $<$ ”, which reads as “is smaller than”;
- “ \subseteq ”, which reads as “is a subset of”;
- “ \in ”, which reads as “is in”;

and many others; note that there is an English verb when you read these symbols out. Since they are verbs, the equations or formulas containing them are treated as sentences, and often preceded by a hidden “that”, making them noun phrases. For instance,

- “We obtain $x + x^2 - y \geq 0$ ” reads as “We obtain that x plus x squared minus y is larger than or equal to 0”.

In some situations they stand as sentences without “that”, as in:

- If $y = x$, then $y > z$.
- The optimizer is nonnegative because $x \in N$.

2.2 Equations and mathematical nouns

If an equation ends a sentence, it must be followed by a full period (.). It may be followed by a comma (,) or nothing if the sentence continues.

Since “that” is usually absorbed into an equation, an equation is often used as a noun phrase, just like other mathematical symbols. Examples:

- Without loss of generality, $x > y$ can be assumed.
- Using the same argument, we can compute $f(x)$.

For the same reason, it is not advisable to write “we have that $x = y$ ”; instead, one should write “we have $x = y$ ”.

In sentences where a “that” clause is led by “it”, omitting “that” would look a bit strange. Examples that do not read well, if not entirely wrong:

- It follows $x + x^2 - y = 0$.
- It is shown $x + x^2 - y = 0$.

One should try to rewrite them. For instance, one can write:

- The condition $x + x^2 - y = 0$ follows.
- It is shown that $x + x^2 - y = 0$ holds.

2.3 “The XYZ theorem” vs “XYZ’s Theorem”

Both phrases are correct. One needs to notice that the use of “the” is very important in the first phrase.

Wrong examples:

- Let us introduce Lebesgue Theorem (missing “the” or “s”).
- Let us introduce the Lebesgue’s Theorem (extra “the” or “s”).

Correct examples:

- Let us introduce Lebesgue’s Theorem.
- Let us introduce the Lebesgue Theorem.

2.4 Displayed fractions

As recommended or required by many journals, fractions in text should appear as “ a/b ” instead of “ $\frac{a}{b}$ ”. The latter should only appear in displayed equations. Similarly, “ $e^{a/b}$ ” usually reads much better than “ $e^{\frac{a}{b}}$ ”. In general, one should try to avoid “tall” formulas in plain text such as matrices and large fractions.

2.5 A function vs the value of a function

Although trivial, sometimes one needs to be reminded that a function is a mapping instead of a value. When a function is defined, always specify its domain, unless it is clear from the context. For a function $f : A \rightarrow B$, $f(x)$ is its value when the argument is $x \in A$. Do not call $f(x)$ a function (it is an element of B and not a mapping), unless, of course, elements of B are themselves mappings, but in that case the meaning of $f(x)$ as a mapping is different from f . On the other hand, $x \mapsto f(x)$ is a function (its domain should be clear from the context). Wrong examples:

- The function $f(x)$ is increasing.
- The function x^2 is differentiable. (Since this expression is used by so many authors and it usually causes no confusion, some believe that it is acceptable; still, it could be ambiguous if x itself is a mapping.)

Correct examples:

- The function f is increasing.
- The function $x \mapsto f(x)$ is increasing.
- The function $x \mapsto x^2$ is differentiable.

Some authors are not strict about this rule when no confusion arises, but I would still recommend it as it makes a clear distinction between a function and its value.

2.6 Use of “ \forall ” and “ \exists ”

These symbols are generally not recommended when writing a mathematical or economic paper. They belong to “blackboard symbols”, i.e., concise symbols to save space and time. In a scientific paper, it is usually not urgent to save space and time with these little symbols. One should write “for all” and “there exists”. If multiple layers of quantifiers are needed together (this may be seen in some areas of computer science), then using “ \forall ” and “ \exists ” may be helpful. For instance, the statement

$$“\forall\theta \in \Theta, \forall x \in X_\theta, \exists\delta \in \Delta_x, \forall z \in Z_\delta, \exists f \in \mathcal{F}, \text{ we have } f(\theta, x, z, n) > \delta”$$

would be too complicated to spell out.

On a related note, it is recommended to write “for all” or “for each/every” instead of “for any” in most places.

2.7 Do not start a sentence with a mathematical symbol

The first letter of a sentence should be capitalized to remind the reader about the proper start of a sentence. If a sentence starts with a mathematical symbol, such a mechanism is lost; that is a reason why it is not recommended. Many journals indeed do not allow the authors to start a sentence with a mathematical symbol. Wrong examples:

- We get $x^2 < 0$. x cannot be a real number.

- $f(x) \geq x^2$ holds.

Correct examples:

- We get $x^2 < 0$. The number x cannot be real.
- We get $x^2 < 0$. Hence, x cannot be a real number.
- It holds that $f(x) \geq x^2$.

2.8 Column vs row vectors

Some journals require vectors to be displayed in bold, as in $\mathbf{x} = (x_1, \dots, x_n)$. This is a good convention to use as long as it does not contradict the practice in the author's field. For column versus row vectors, I use the following rule. It is a personal suggestion and I hear some objections.

All vectors are column vectors. There are no row vectors in mathematics.

In other words, a vector \mathbf{x} can only be interpreted as an $n \times 1$ matrix when it is used as a matrix, and a so-called "row vector" is a $1 \times n$ matrix and not a vector. Certainly, such a rule matters only when matrix operations are involved, and one can freely view vectors as either rows or columns in a context without matrix operations (for instance, when working with sequences or tuples).

This rule has several advantages. Below, \mathbf{x} is an n -dimensional vector where $n \geq 2$, and A is an $n \times n$ matrix.

1. This rule gives a unique way to use vectors in matrix operations and thus minimizing confusion.
2. Even without context, if one sees something like $\mathbf{x}^\top A \mathbf{x}$ or $\mathbf{x}^\top \mathbf{x}$, one immediately knows that it represents a constant, because \mathbf{x} can only be column vector. On the other hand, $\mathbf{x} \mathbf{x}^\top$ and $\mathbf{x} A \mathbf{x}^\top$ are immediately $n \times n$ matrices.
3. This rule allows us to put a linear transform A in front of the vector \mathbf{x} being transformed; that is, $A \mathbf{x}$ is the transformed vector. If \mathbf{x} is interpreted as a row vector, then a transformed vector needs to be $\mathbf{x} A$ or $(A \mathbf{x}^\top)^\top$. A system of linear equations can be written as $A \mathbf{x} = \mathbf{y}$.

4. A common confusion arises as the form $\mathbf{x} = (x_1, \dots, x_n)$ displays the components in a row. I believe that this confusion is due to not paying enough attention to the commas. Note that a $1 \times n$ matrix with entries x_1, x_2, x_3 , can be written as $(x_1 \ x_2 \ x_3)$. Please notice the difference caused by the commas: They break the rows. This is an advantage of the rule, because most mathematicians would agree that (x_1, x_2, x_3) and $(x_1 \ x_2 \ x_3)$ should not represent the same thing, as commas are not negligible in either English or mathematical writing.
5. It is consistent with the convention in linear systems of equations. For instance, the equality $(x_1, x_2, x_3) = (y_1, y_2, y_3)$ should be understood formally as

$$x_1 = y_1$$

$$x_2 = y_2$$

$$x_3 = y_3$$

although we use the short-hand notation $(x_1, x_2, x_3) = (y_1, y_2, y_3)$.

2.9 Use of “denote”

The verb “denote” is the action to be a symbol. The subject is the symbol, and the object is the mathematical quantity to be denoted. Wrong examples:

- Denote $x = f(0)$.
- Denote x as the value $f(0)$.

Correct examples:

- Denote the value $f(0)$ by x . The value $f(1)$ is denoted by y .
- Denote by x the value $f(0)$. Let y denote $f(1)$.

2.10 Tall mathematical objects inside brackets

When writing a tall mathematical object (such as a fraction) inside brackets (parentheses) in displayed equations, one should always try to make the parentheses large enough. Inappropriate examples:

$$\mathbb{P}(e^{X^2} > \frac{1}{5}); \quad \mathbb{E}[\frac{X+Y}{X^2+Z^2}]; \quad \inf\{f(\frac{1}{e^z+1}) : z > 0\}; \quad |\frac{x+1}{2}|; \quad \lfloor \frac{y}{n} \rfloor.$$

Good examples:

$$\mathbb{P}\left(e^{X^2} > \frac{1}{5}\right); \quad \mathbb{E}\left[\frac{X+Y}{X^2+Z^2}\right]; \quad \inf\left\{f\left(\frac{1}{e^z+1}\right) : z > 0\right\}; \quad \left|\frac{x+1}{2}\right|; \quad \left\lfloor\frac{y}{n}\right\rfloor.$$

The above formulas are written by using L^AT_EX commands `\left` and `\right` which automatically adjust the size of parentheses. In some situations, the automatic size may not be ideal, and authors may manually choose the size for parentheses.

2.11 Use of \sum , \prod , \cup , \vee versus $+$, \times , \cup , \vee

Some mathematical operations, such as \sum , \prod , \cup and \vee , are meant to be applied to a set of objects, whereas their corresponding bivariate operations $+$, \times , \cup and \vee are meant to be applied strictly to two objects. These two ways of using operations are *not interchangeable*. For instance, one can write

$$\sum_{i \in \{1,2\}} x_i, \quad \sum_{i \in \{1,2\}} x_i, \quad \text{or} \quad x_1 + x_2,$$

but one should not write

$$x_1 \sum x_2, \quad x_1 \sum x_2, \quad \text{or} \quad +_{i \in \{1,2\}} x_i.$$

The same applies to the other operations. This looks trivial, but I often observe the error of writing \cup for \bigcup , that is, writing $\cup_{i \in B} A_i$ when it should be $\bigcup_{i \in B} A_i$.

The set-input operations will put the qualifier, “ $i \in \{1, 2\}$ ” in the above example, underneath the operation in displayed equations and put it as a subscript in in-text formulas.

2.12 Maximum, maximizer, maximal, and greatest

The words “maximum”, “maximizer”, “maximal”, and “greatest” have different meanings in mathematics, and they should not be used interchangeably.

When it comes to a real function, the maximum refers to the largest value it can take, and the maximizer refers to the point at which the largest value is taken. That is, for the function $f : \mathbb{R} \rightarrow \mathbb{R}$, $x \mapsto 1 - x^2$, the maximum of f on \mathbb{R} is 1 and the maximizer of f on \mathbb{R} is 0.

When it comes to a set with an ordering structure, a maximal element is one that is not dominated by any other element, and a greatest (or largest) element is one that dominates

all other elements (it is often unique). In this context, a greatest element is also called a maximum, and the word “maximizer” should not be used.

3 On grammar and choices of words

3.1 Connecting two clauses

There are two common ways to connect two clauses; one is using a *conjunction* like “and”, “or”, “but”, and the other is to use a semicolon (;).

Wrong examples:

- We define the function f by $f = g^2$, it has nice properties.
- Some results are available, see Wang (2020).

Correct examples:

- We define the function f by $f = g^2$, and it has nice properties.
- Some results are available; see Wang (2020).

This also applies to equation sentences; for instance, “ $x = 1$ and $y = 2$ ” is correct, but “ $x = 1, y = 2$ ” is not.

3.2 Use of “then”

The word “then” leads to some common mistakes. This word cannot connect two clauses because it is not a conjunction. Usually, “then” should be either used together with “if” for a logical relation or used to describe a temporal relation. For instance, the following sentence is correct:

- The paper was written in 1920. Back then, statisticians did not have computers.

Wrong examples:

- Let $y = 5$, then $y > 0$.
- We first specify x , then we specify y .

Correct examples:

- If $y = 5$, then $y > 0$.
- We first specify x , and then we specify y .

As a rough rule of using “then” in mathematical logic, “then” should be used in sentences which would still be grammatically correct after removing “then”.

Another place where I would advise against “then” is to use it to start a sentence. Many authors start a sentence with “then” in mathematical theorems. For instance, it may be common to see

- **Theorem.** Suppose that f satisfies condition (1). Then condition (2) holds.

I understand that sometimes it becomes helpful (as in the above example), although I would try to avoid it in most situations.

3.3 Use of “when”

The word “when” cannot replace “if”; if replacing “when” by “if” does not change the intended meaning of your sentence, you should not use “when”. Wrong examples:

- When $n = 1$, we have $f(n) > g(n)$.
- Condition A holds when x is taken from \mathcal{X} .

Correct examples:

- If $n = 1$, we have $f(n) > g(n)$.
- Condition A holds if x is taken from \mathcal{X} .

I usually would advise against use of “when” except for describing a temporal relation. For instance, the following is correct:

- When the paper was written, statisticians did not have computers.

3.4 Use of “it”

A common situation observed from students’ writing is that they often write “it means that ...” without clearly indicating what “it” refers to. When “it” appears, the word should always clearly represent a previously mentioned noun or idea, or one to be mentioned, as in “it follows **that** ...” or “it is good **to** ...” where the bold rephrase or clause is represented by “it”. A similar confusion can arise when one writes “this implies ...” without clearly indicating what “this” refers to.

3.5 Use of apostrophes as in “it’s” or “don’t”

One should always spell out these colloquial phrases; we are not in a rush to finish the sentence. The apostrophe (’) should not be used in a written scientific article, with a few exceptions. The main exception is that one can use a possessive (’s). Wrong example:

- Let’s introduce Lebesgue’s Theorem.

Correct example:

- Let us introduce Lebesgue’s Theorem.

3.6 Writing “cannot”

This word should always be spelt as “cannot” instead of “can not” or “can’t” (a common mistake for nonnatives).

3.7 Use of “the” in journal and university names

Using the articles “the” and “a” properly is a challenging task for people whose native language does not have such a grammatical object. Rules on using these articles can be found online everywhere and will not be explained here.

I am only making one point specific to academic writing. If a university or a journal (often with “of”) is mentioned *in a sentence*, it should always have “the”, such as “the University of Oxford”, “the University of Waterloo”, “the Massachusetts Institute of Technology”, “the Journal of Finance”, and “the Annals of Statistics”.

University names without “of” usually do not have “the”, such as “Harvard University”, “Yale University”, “Northwestern University”, and “Peking University”. An exception to the above rule is “the George Washington University”. Please check the official website of the university for the correct usage.

The first “the” *may be omitted* if it appears at the beginning of the *title* of a book, a paper or a section, or the headline of a newspaper, such as

- “Theory of Games and Economic Behavior” (Von Neumann and Morgenstern, 1947).

“Empirical analysis” would be a suitable section title, although in a sentence one should say “the empirical analysis” or “an empirical analysis”. That is why one often sees “Journal of Finance” and “University of Oxford” without “the” in their title. However, in some situations

it would be unacceptable to omit the first “the” in a title, as in, for instance, “The Old Man and the Sea” (Hemingway, 1952).

Another situation where “the” may be omitted is when it is absorbed into an abbreviation. For instance, “in the Department of Mathematics at MIT” is as good as “in the Department of Mathematics at the Massachusetts Institute of Technology”. Moreover, one should correctly use “in” and “at” in the above phrases.

3.8 Use of hyphen (-) in compound adjectives

The rough rule for hyphenating compound adjectives (also called phrasal adjectives) can be summarized as follows.

- If a compound adjective is used in front of the modified noun, then a hyphen should be used.
- If a compound adjective is used after the modified noun, then a hyphen should not be used.

Some examples:

- “We assume an arbitrage-free market” and “We assume that the market is arbitrage free”.
- “They cited an in-press paper” and “The cited paper is in press”.
- “The mapping is a translation-invariant risk measure” and “The risk measure is translation invariant”.

There are exceptions to this rule, for instance, phrases like “award-winning”, “cost-effective”, “old-fashioned”, “short-lived”, “time-tested”, etc., are usually hyphenated in all places. Please search online for details when in doubt.

3.9 Capitalization after a colon

In titles and headings (including section headings), the initial letter after a colon should always be capitalized, even if all other words are not capitalized. For instance,

- Belief hedges: Measuring ambiguity for all events and all models (Journal of Economic Theory, Volume 198, December 2021).

In sentences, the initial letter after a colon should be capitalized if it introduces a full sentence or several full sentences (APA style). When introducing a list of objects (nouns), then the initial letter should not be capitalized. Examples:

- At that time, Einstein was right: Such a development was needed in physics.
- I took several courses in natural sciences in my freshman year: three courses in mathematics, one in physics, and two in chemistry.

It should be fair to mention that some authors do not agree to capitalize the first letter in a sentence after a colon. On this matter, there is a disagreement between British English and American English. The Chicago Manual of Style also has a slightly different rule from APA.

3.10 Use of “thus”, “hence”, and “therefore”

These three words cannot connect two clauses as they are not conjunctions. Wrong example:

- We get $x = 1$, hence $x > 0$.

Correct examples:

- We get $x = 1$, and hence $x > 0$.
- We get $x = 1$. Hence, $x > 0$.

My personal suggestion is not to use “thus” too often, although it is not incorrect to use it.

3.11 Using the Oxford comma

An Oxford comma is the final comma used in a list of more than two objects, as in the last comma in “A, B, and C”. If the meaning is clear, the Oxford comma may or may not be used, but usually we skip it among mathematical objects (for instance, “1, 2 and 3”). When in doubt, one should include the Oxford comma.

In recent years, there is a tendency to include the Oxford comma in relatively long titles in economics and finance. For instance, a few papers in 2022:

- Public debt, consumption growth, and the slope of the term structure (Review of Financial Studies, Volume 35, August 2022)
- Terrorism financing, recruitment, and attacks (Econometrica, Volume 90, July 2022)

- Financial crisis, creditor-debtor conflict, and populism (Journal of Finance, Volume 77, May 2022)

3.12 Plural vs singular nouns

There are several nouns that are commonly not written in plural form.

1. “work”: We write “the work of someone” instead of “the works of someone”, even if we are referring to multiple papers.
2. “research”: This word does not have a plural form “researches”; the latter is a common mistake.
3. “notation”: This word should usually be singular, e.g., “the notation of the paper”. An exception occurs in the context where multiple sets of notations exist.
4. “literature”: This word should usually be singular, e.g., “the finance and insurance literature”. If we say “the finance and insurance literatures”, we really emphasize that the two streams of studies are separate.

3.13 “First” vs “firstly”

The word “first” is perfectly fine as an adverb. The general recommendation is that one should always use “first”, “second”, etc., instead of “firstly”, “secondly”, etc. There are some places where both seem to be acceptable, and there are many places where only “first” is acceptable. I do not know any place in which “firstly” is preferred.

3.14 Singular “they”

When referring to an unspecific person (e.g., a general “agent” in an economic study) in academic writing, the recent media recommended to use “they/their” or “she/her”. For instance, APA writing style encourages to use the singular “they”: <https://apastyle.apa.org/style-grammar-guidelines/grammar/singular-they> (accessed in August 2022; language is evolving). To avoid confusion, if possible, avoid “they are” when referring to only one agent. However, “their” is fine, such as:

- Each agent optimizes their portfolio.
- An investor never reveals their true preference.

3.15 Prepositions following “bound”

When referring to a mathematical statement on an upper bound or a lower bound, always use “bound on X ” where X is the quantity with the specified bound. The preposition “on” should not be replaced by “for” or “of”. However, when X is not the quantity with the bound but something else, the preposition should not be “on”. Wrong example:

- We obtain that g is an upper bound for f .

Correct examples:

- We obtain that g is an upper bound on f .
- We obtain that g is an upper bound (omitting “on f_θ ”) for each θ .

3.16 Use of “i.e.” and “e.g.”

When using “i.e.” (standing for Latin “*id est*”, meaning “that is”) and “e.g.” (standing for Latin “*exempli gratia*”, meaning “for example”), it is recommended that they should be followed by a comma. Examples:

- Let x be 5; i.e., x is the largest value that y can take.
- Let x be 5, i.e., the largest value that y can take.

Also note above that the semicolon (;) should be used before “i.e.” when it introduces a sentence, and the comma (,) should be used before “i.e.” when it introduces a noun rephrase.

Some authors discourage the use of “i.e.” and “e.g.” in scientific writing.

3.17 Using hyphens, en dashes, em dashes, and minus signs

Hyphens (-), en dashes (–), and em dashes (—) are *not interchangeable*. This difference may not exist in some other languages, making it harder to notice for some authors.

- Hyphens are used to join words to form a compound word. Examples: “a mean-preserving spread”, “an n -dimensional ball” and “a σ -additive function”.
- En dashes are used to show a range or to connect two things. Examples: “the Black–Scholes formula”, “the put–call parity”, “the 2007–2009 financial crisis”, and “pages 1–10”. I hear some controversy on whether an en dash or a hyphen should be used in “Black–Scholes” above.

- Em dashes are used to indicate a break in thought, to introduce a new idea, or to emphasize a point. Examples: “Four axioms—completeness, transitivity, continuity, and independence—were introduced by von Neumann and Morgenstein”. Whether to put spaces around em dashes is a matter of style but most styles recommend using em dashes without spaces on either side.

In \LaTeX , hyphens, en dashes, and em dashes are produced by, respectively, typing one, two, and three consecutive hyphens (-).

The hyphen in the \LaTeX mathematical environment ($\$...\$$) becomes a minus sign as in “ $x - y = 0$ ”, which looks very different from a hyphen. A common mistake is to write “ n -dimensional” or “ σ -field” (with “-” inside $\$...\$$), which should be “ n -dimensional” or “ σ -field” (with “-” outside $\$...\$$).

3.18 Use of “that” and “which”

When it comes to heading relative clauses, the words “which” and “that” have different meanings and different ways of usage. A very common mistake in writing of students is to use them interchangeably, especially when it comes in the second half of a sentence.

The word “which” indicates a nonrestrictive clause, and it should always follow a comma (,) whereas the word “that” indicates a restrictive clause, and it should not follow a comma. Examples:

(a) The set A has two elements which are positive.

- This sentence is incorrect and has no clear meaning. (If I see such a mistake in written papers, my first guess is that the authors mean (b) below, but I will carefully check the surrounding text for assurance.)

(b) The set A has two elements, which are positive.

- This sentence means the following: “The set A has two elements. They are positive.” In general, “which”-clauses can be rewritten as separate sentences without changing the meaning.

(c) The set A has two elements that are positive.

- This sentence means that precisely two elements of A are positive, but A may have other elements that are not positive.

Clearly from the above examples, the sentence with “which” and the one with “that” have different mathematical meanings.

When it comes to “who”, it is even trickier, as this word can take both the positions of “which” and “that”. The comma is even more crucial in such a situation. Examples:

- The co-authors, who are professors at MIT, did not respond.
 - This sentence means that all co-authors did not respond, and they were all professors at MIT.
- The co-authors who are professors at MIT did not respond.
 - This sentence means that only those professors at MIT did not respond, and other co-authors may have responded, and they may not be professors or at MIT.

4 On citation

4.1 Citing a paper or not

A general suggestion is to cite a paper only for four purposes.

- The first is attribution; that is, the cited paper has made a contribution to the topic that cannot or should not be neglected, or it is intimately linked to the current paper’s study. In this case, the reader may not need to read the cited paper; simply acknowledging the existence of it is the authors’ purpose.
- The second is context support; that is, the cited paper should be able to help the reader to understand or be convinced by the current paper. In this case, the reader is advised to read the cited paper, and otherwise some statements in the current paper may not be clear or well supported. Citing a theorem or a result is of this type, and so is citing an opinion, an example or a dataset.
- The third is contracting; that is, the cited paper poses a disagreement with the current paper and it would be unfair not to compare, discuss, and justify the current paper’s approach. This can sometimes be brought up by referees in a review process.
- The paper is of importance, relevance and interest, although it does not fit the above three categories.

Generally, if both the following statements hold:

- you do not truly mean to say that the cited paper is of great importance and relevance to the topic;
- you do not truly mean to advise the reader of your paper to read the cited papers,

then you should not cite them. I observe this malpractice often when a laundry list of references appears. It happens more often in recent years as papers tend to have longer and longer reference lists. I believe this should be avoided as much as possible.

4.2 Citing an old paper or a paper in a different language

When an old paper is cited (e.g., those which are 50 years old), one often does not really mean to urge the reader to read the original paper. In most cases, old papers were written for people with a different knowledge set than current readers, so it is often not advisable for the reader to directly read the old papers. If the cited old paper is a classic masterpiece, it most likely has a modernized version in a more recent book or review paper, which is better for the reader to read. For instance, “See Einstein (1905) for this theory” would be a strange sentence to appear in 2022; a better sentence is “This theory was studied by Einstein (1905)” as this does not invite the reader to read the original text. One could add “A recent review is given by ...” if further reading on the topic helps to understand the current paper. The same applies to papers written in a different language than the current paper.

This suggestion is not meant to discourage one from citing old papers, but to encourage citing them with the help of a modern access.

4.3 Citing a book

When citing a book for a result, make sure to identify which result/page/section should be cited. For instance, “As shown by Hardy et al. (1934), the average of X is less than the average of Y ” is a poor sentence, because the book of Hardy, Littlewood and Pólya (1934, *Inequalities*) contains hundreds of theorems. A better sentence is “Using Hardy et al. (1934, Theorem 16), the average of X is less than the average of Y ”.

Similarly, unless referring to a general treatment of a topic, as in “For a comprehensive treatment on this topic, see X ”, one should try to give specific section or page numbers. For instance, “See X for discussions of this result” is uninformative if X is a 500-page book, but it would be fine if X is a regular-sized paper.

4.4 Referring to the authors of a paper

If a paper is cited in the “author-year” format, then it may refer to either the paper or the authors. For example, “Wang and Zhang (2020)” may refer to the paper or the two authors. My personal recommendation is to treat it as the authors by default. Following this line of recommendation, it is not advisable to write “Wang and Zhang (2020) contains a dataset”, but rather to write “Wang and Zhang (2020) provided a dataset”. However, this is a bit different when the citation style “[x]” is used; it is fine to write “The paper [1] contains a dataset”. Another exception is that when one writes “see Wang and Zhang (2020)”, it clearly refers to the paper instead of the authors.

4.5 Citing a result from a paper

Always use “of” when you refer to a result of some authors, as in “Theorem 1 of Wang and Zhang (2020)”, instead of “Theorem 1 in Wang and Zhang (2020)”. Again, if the citation style “[x]” is used, then the situation is different; one can say “Theorem 1 in [x]”. Some scholars prefer using “by”, as in “Theorem 1 by Wang and Zhang (2020)”.

5 On typesetting

5.1 Aligned equations

In aligned equations, mathematical verbs should only be aligned with mathematical verbs. For instance, “=” should be aligned with “≥”, but “=” cannot be aligned with “+”; an example is provided below:

$$\begin{aligned} f(x) &= (x+1)^2 - \log(1+x) + e^x \\ &\geq x^2 + 2x + 1 - x \\ &\quad + 1 + x + x^2 + \cdots + x^n, \quad \text{for } x > 0. \end{aligned}$$

The one below reads bad:

$$\begin{aligned} f(x) &= (x+1)^2 - \log(1+x) + e^x \\ &\geq x^2 + 2x + 1 - x \\ &\quad + 1 + x + x^2 + \cdots + x^n, \quad \text{for } x > 0. \end{aligned}$$

5.2 Equation length

Equations should never be longer than the width of the text. You can define additional symbols to substitute some longer parts of the equation if needed.

5.3 The dot (.) in L^AT_EX text should be followed by a tilde (~)

The tilde (~) in the L^AT_EX code appears in the generated pdf file *as a space*. The reason of using the tilde instead of a usual space after a dot (as in “Dr. Wang”) is that L^AT_EX by default adds some space after the end of a sentence, and it treats a sentence as ended if it sees a dot/period followed by a space (or by a closing parenthesis and then a space). Observe the subtle difference in spacing between

“e.g. Prof. Dr. R. Wang” (with the tilde) and

“e.g. Prof. Dr. R. Wang” (without the tilde).

(Here, if “e.g.” is followed by a comma as recommended in Section 3.16, then the unintended extra spacing following “e.g.” is automatically avoided.)

Sometimes this spacing difference can appear quite big depending on the surrounding texts. Words connected with a tilde will not be broken across lines. An alternative which allows for line break is to use a backslash-space (\) in place of a tilde.

5.4 Numbering your equations

Equations should only be given a number if they are referred to in another part of the paper. (I observe that some scholars do not agree with this convention.) On the other hand, all theorems, lemmas, examples, remarks, etc, should be numbered.

The L^AT_EX package `refcheck` can check unused and unlabelled equations and bibliographic items.

5.5 Using footnotes

When a footnote is used after a term followed by a punctuation mark such as a period (.) or a comma (,), the convention is that the footnote should follow the punctuation mark. Example:

- The paper was published in 2020.¹

¹A preliminary version of the paper was posted online in 2018. Observe that this footnote should be put after the period.

5.6 Writing mathematical dots in \LaTeX

In the \LaTeX mathematical environment, there are three similar commands for dots: `\dots`, `\ldots` and `\cdots`. In most situations, it is recommended to use `\dots`, which automatically adjusts its position according to the surrounding objects. For instance, “ X_1, \dots, X_n ” and “ $X_1 + \dots + X_n$ ” are both produced by using `\dots`.

5.7 Space after and before parentheses

When writing parentheses in text, there should be a space before and other any set of parentheses. The mistake of missing the space is common for writers whose native language does not have spaces.