An Axiomatic Theory for Rating Structured Finance Securities

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Agenda

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Structured finance securities

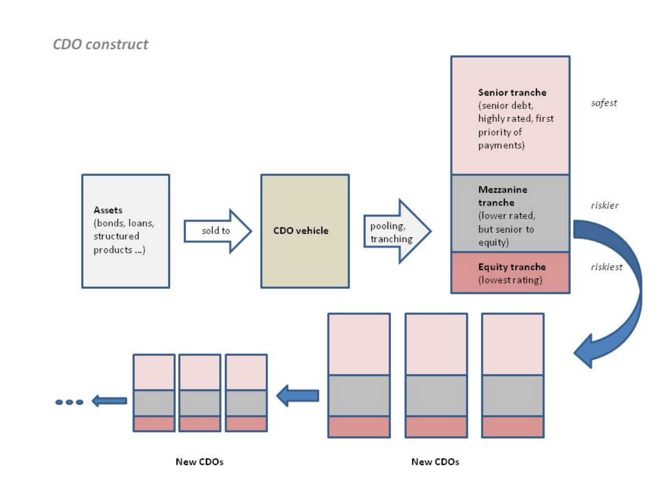
Two-step initialization of structured finance securities

- pooling financial assets, such as corporate bonds, auto loans, and mortgages, into a large portfolio
  - a Special Purpose Vehicle (SPV)
- tranching the portfolio into sequential classes of securities
  - e.g. CDOs

A key goal of the structuring process is

- to create at least one class of securities whose rating is higher than the average rating of the underlying collateral pool.
- Reason: some investors are happy to hold a speculative grade bond, while most seek safer bonds.

CDOs: an example



Credit ratings

- Credit ratings are categorical characteristics of defaultable securities (bonds)
  - AAA, AA, A, BBB, ...
- Investors rely heavily on credit ratings as a basis for pricing and risk management

Primary examples.

- Standard and Poor's (S&P) and Fitch use the probability of default (PD) as their primary rating factor
- Moody's uses the expected loss (EL) as the primary rating factor

General settings for rating criteria

Some basic components

- A probability space $(\Omega, \mathcal{F}, \mathbb{P})$
- $\mathcal{L}^\infty$ is the set of bounded random variables; $\mathcal{L}_{[0,1]}$ is the set of $[0,1]$-valued random variables
- The set $\mathcal{X}$ of all possible "bonds"

$$\mathcal{X} = \{(L, M) \in \mathcal{L}^\infty \times \mathbb{R}_+ : 0 \le L \le M\};$$

- (L, M) represents
  - asset pools, tranches, defautable bonds, ...
  - loss L and nominal value M
  - with "similar" maturities (one-period)

General settings for rating criteria

Definition

A rating criterion is $\mathcal{I} : \mathcal{X} \to \{1, \ldots, n\}$ satisfying

[SI] Scale invariance: $\mathcal{I}(\lambda L, \lambda M) = \mathcal{I}(L, M)$ for all $(L, M) \in \mathcal{X}$ and $\lambda > 0$.

Write $I_k = \{(L, M) \in \mathcal{X} : \mathcal{I}(L, M) = k\}$, $k = 1, \ldots, n$.

- I_1 is the best rating (e.g. AAA), I_n is the lowest rating (e.g. D)
- For $(L, M) \in \mathcal{X}$, it is sufficient to consider $L/M \in \mathcal{L}_{[0,1]}$

Primary examples

Example

- The PD criterion: For some $p_0 < 0 < p_1 < \cdots < p_n = 1$,

$$I_k = \{(L, M) \in \mathcal{X} : \mathbb{P}(L > 0) \in (p_{k-1}, p_k]\}.$$

- The EL criterion: For some $q_0 < 0 < q_1 < \cdots < q_n = 1$,

$$I_k = \{(L, M) \in \mathcal{X} : \mathbb{E}[L/M] \in (q_{k-1}, q_k]\}.$$

Key question

PD, EL, or another?

Large literature (and many recent) on risk measures

- VaR versus ES, or others (Basel III/IV, Solvency II, SST, ...)
- Mathematical considerations
  - modeling, optimization, computation, complexity, ...
- Statistical considerations
  - uncertainty, robustness, backtesting, inference, ...
- Economic axioms (e.g. Artzner-Delbaen-Eber-Heath'99 (MF))

Limited or no literature on axiomatic approach for rating criteria?

Progress

The Bloomberg CDO database

- Rating coverage of all CDOs listed on Bloomberg (in US$), all rated by S&P, Moody's or Fitch
- Issuance dates from January 1997 to December 2018
- The Dodd-Frank Act was passed on July 2010
  - The pre-Dodd-Frank period: 1,782 deals ($0.92 trillion)
  - The post-Dodd-Frank period: 1,792 deals ($1.29 trillion)

Rating coverage

Panel A: Deal-level rating coverage

	Before Dodd-Frank			After Dodd-Frank		
	Number	Capital ($B)	% capital	Number	Capital ($B)	% capital
Solo rating	170	56.3	5.9	283	159.0	12.3
S&P	61	15.9	1.7	112	51.4	4.0
Moody's	81	33.8	3.5	167	102.5	7.9
Fitch	28	6.6	0.7	4	0.2	0.4
Multiple ratings	1612	903.3	94.1	1509	1130.8	87.7
SP & Moody's	1189	680.2	70.9	686	528.5	41.0
SP & Fitch	68	24.0	2.5	141	95.8	7.4
Moody's & Fitch	41	19.4	2.0	673	497.8	38.6
S&P, Moody's and Fitch	314	179.7	18.7	9	8.6	0.7
Panel B: Trache-level rating coverage						
AAA rated Tranches	3434	674.4	79.5	2733	530.6	65.8
non-AAA rated Tranches	6522	173.8	20.5	8015	275.5	34.2
Total	9956	848.2	-	10748	806.1	-

Numbers of deals dual related by S&P and Moody's

	Year	S&P non-AAA	Moody's non-AAA	Both non-AAA
Before crisis	1997	0	0	0
	1998	3	1	1
	1999	9	0	3
	2000	2	10	16
	2001	0	14	35
	2002	1	2	69
	2003	0	5	65
	2004	1	5	99
	2005	0	1	188
	2006	1	9	402
	2007	4	9	335
Crisis	2008	0	1	30
	2009	0	1	0
	2010	0	0	6
After Dodd-Frank	2011	23	0	6
	2012	82	3	18
	2013	127	13	15
	2014	84	23	10
	2015	33	18	16
	2016	33	28	4
	2017	35	17	34
	2018	37	5	21

Numbers of deals grouped by numbers of tranches

Distribution of deal numbers dual related by S&P and Moody's

Number of tranches	Before Dodd-Frank S&P non-AAA	Before Dodd-Frank Moody's non-AAA	Before Dodd-Frank Both non-AAA	After Dodd-Frank S&P non-AAA	After Dodd-Frank Moody's non-AAA	After Dodd-Frank Both non-AAA
2	3	15	31	3	1	1
3	2	15	123	10	5	4
4	14	22	311	4	8	8
5	1	2	459	265	62	39
6	1	2	199	166	30	75
7	0	2	65	6	1	0
8–13	0	0	54	0	0	0
Total	21	58	1242	454	107	127
Mean	3.762	3.431	4.908	5.319	5.103	5.441
Std.	0.944	1.216	1.396	0.688	0.812	0.813

Number of tranches and rating methods

	Before Dodd-Frank		After Dodd-Frank	
	(1)	(2)	(3)	(4)
S&P non-AAA	0.415	0.571	0.429	0.217
	(1.052)	(1.432)	(5.982)	(2.897)
Moody's non-AAA	-0.321	-0.124	0.125	-0.02
	(-1.424)	(-0.484)	(1.171)	(-0.193)
Both non-AAA		0.359		0.277
		(2.372)		(2.777)
Solo S&P		-1.1414		-1.583
		(-4.529)		(-12.417)
Solo Moody's		-1.2545		-0.996
		(-5.520)		(-7.96)
Deal size	0.711	0.559	0.249	0.042
	(18.277)	(13.682)	(5.095)	(0.850)
Collateral controls	y	y	n	n
Year controls	y	y	y	y
Issuer controls	n	n	y	y
No. Obs.	1782	1782	1792	1792
R^2	0.320	0.357	0.608	0.649

Summary of empirical observations

- S&P and Moody's are the main raters
- For non-AAA tranches, after Dodd-Frank:
  - non-AAA tranches increase in terms of capital percentage
  - dual rating almost disappears
  - S&P tends to be more attractive to issuers
  - S&P tends to generate more tranches
- Question: Is there a theoretical explanation of the empirical observations, especially after Dodd-Frank?

Related literature

- Credit shopping and credit catering between agencies: Fender-Kifff'05 (JCR), Griffin et al.'13 (RFS)
- Rating arbitrage: Hull-White'12 (JDer)
- Gains from tranching: Brennan et al.'09 (EFM)
- Critiques of credit ratings: Coval et al.'09 (AER), Wojtowicz'14 (JBF), Cornaggia-Cornaggia'13 (RFS), Cornaggia et al.'17 (RoF)
- Choquet integrals: Yaari'87 (ECMA), Kou-Peng'16 (OR), W.-Wei-Willmot'19 (MOR)
- (Systemic) risk measures: Chen et al.'13 (MS), Cherny-Madan'09 (RFS), Acharya et al.'12 (AER), Acharya et al.'17 (RFS)
- Scenario-relevance: W.-Ziegel'18

Progress

Subjective prices

Rating criteria are a pricing reference for both investors and issuers

- We need to connect the two considerations

How does rating affect prices?

- Ashcraft-GoldsmithPinkham-Hull-Vickery'11 (AER) on MBS:
  - study: causal effect of ratings on security prices
  - result: "MBS prices are *excessively* sensitive to credit ratings, relative to the informational content of ratings."
- We assume an idealistic mathematical world:
  - issuers use rating as a prediction of average security prices
  - investors use ratings as a reference for pricing

Subjective prices

Definition

A functional $p : \mathcal{X} \to \mathbb{R}_+$ is called a (subjective) price of defaultable bonds if it satisfies

- $p(L_1, M) \geq p(L_2, M)$ for all $(L_1, M), (L_2, M) \in \mathcal{X}$ with $L_1 \leq L_2$;
- $p(\lambda L, \lambda M) = \lambda p(L, M)$ for $\lambda \in \mathbb{R}_+$ and $(L, M) \in \mathcal{X}$.

- Each investor/issuer may have their own subjective price

Subjective prices

Connecting ratings and prices

- A subjective price p is compatible with $\mathcal{I}$ if for all (L_1, M), $(L_2, M) \in \mathcal{X}$,

$$\mathcal{I}(L_1, M) < \mathcal{I}(L_2, M) \Rightarrow p(L_1, M) > p(L_2, M). \quad (1)$$

- p is strictly compatible with $\mathcal{I}$ if "$\Rightarrow$" in (1) holds as "$\Leftrightarrow$".

"Higher rating, higher price"

- Investors use rating as a reference $\Rightarrow$ compatibility
- Issuers use rating as a prediction $\Rightarrow$ strict compatibility

Subjective prices

Trade-off between compatible and strictly compatible prices

- A compatible price can take continuous values, more flexible
- A strictly compatible price only takes discrete values for bonds with nominal 1
- Information asymmetry ⇒ a market for lemons[1] ⇒ strictly compatible prices are reasonable approximations of market prices

[1] Downing-Kaffee-Wallace (2009 RFS)

Tranche exploitation

Tranching schemes:

- A defaultable bond $(L, M) \in \mathcal{X}$
- An issuer issues m tranches of (L, M)

A tranching scheme of (L, M) is a vector $(K_1, \dots, K_m)$

- Each K_j is a tranche level, $M > K_1 > \cdots > K_{m-1} > K_m = 0$
- (L, M) itself is a trivial tranching scheme (0) with $m = 1$
- The j-th tranche is $((L - K_j)_+ \wedge (K_{j-1} - K_j), K_{j-1} - K_j)$ where $K_0 = M$.

Tranche exploitation

- For a subjective price p, the portfolio value of the tranching scheme $(K_1, ..., K_m)$ is

$$\sum_{j=1}^{m} p((L - K_j)_+ \wedge (K_{j-1} - K_j), K_{j-1} - K_j).$$

- To get a higher total value, the issuer tries to maximize the above value over m and $(K_1, \dots, K_{m-1})$.

Tranche exploitation

For a given rating criterion $\mathcal{I}$:

- A tranching scheme is maximal if it has the maximum number of distinct rating categories among all tranching schemes of the same bond

Definition

$\mathcal{I}$ leads to tranche exploitation for a subjective price p, if for all $(L, M) \in \mathcal{X}$, a maximal tranching scheme strictly dominates all non-maximal tranching schemes in value.

- $\mathcal{I}$ leads to tranche exploitation for $p \Rightarrow$ the issuer (using p) issues as many tranches of different credit ratings as possible

Tranche exploitation

Proposition

The PD criterion leads to tranche exploitation for all strictly compatible prices, and for $n \geq 3$, the EL criterion does not lead to tranche exploitation for any strictly compatible prices.

- The PD criterion as pricing reference $\Rightarrow$ excessive issuance of tranches, regardless of the actual pricing scheme used
- Partly explains the empirical observations

Tranche exploitation

Is tranche exploitation a bad thing?

- Does a tranching scheme increase the overall value of the asset pool?
- In the spirit of the MM Theorem[2], the value of the collateral pool and that of the tranches should be equal
    - An asset pool is an SPV, usually a limited company
- Investors (as a whole) pays more than they should

[2] Modigliani-Miller'58 (AER)

Progress

Balanced prices

Motivated by the MM Theorem

Definition

A subjective price $p : \mathcal{X} \to \mathbb{R}_+$ is balanced if it satisfies

$$p((L-K)_+, M-K) + p(L \wedge K, K) = p(L, M)$$

for $(L, M) \in \mathcal{X}$ and $K \in [0, M]$.

- A tranching scheme does not change the value of the portfolio
- A sophisticated investor's price is balanced
- A reasonable rating criterion should be acceptable for some sophisticated investors

Self-consistency

The first axiom

[SC] Self-consistency: there exists a balanced subjective price compatible with the rating criterion $\mathcal{I}$.

Proposition

The EL criterion is self-consistent, and, for $n \geq 3$, the PD criterion is not.

Self-consistency and tranche exploitation:

Theorem

Assume $n \geq 3$. A self-consistent rating criterion does not lead to tranche exploitation for any strictly compatible subjective price.

Economic scenario relevance

For a collection of scenarios $S = (S_1, \dots, S_m)$ and $X, Y \in \mathcal{L}^\infty$ write $X \overset{S}{\sim} Y$ if $X \overset{d}{=} Y$ on S_j for each $j = 1, \dots, n$.

[SR] Scenario relevance (with respect to S): $\mathcal{I}(L_1, M) = \mathcal{I}(L_2, M)$ for all $(L_1, M), (L_2, M) \in \mathcal{X}$ satisfying $L_1 \overset{S}{\sim} L_2$.

- If S is a constant, [SR] reduces to the standard property of law-invariance [LI]
- [LI]⇒[SR]
- Both the PD and the EL criteria satisfies [LI] and [SR]
- Scenario-based risk measures (W.-Ziegel'18)

Characterization theorems

Definition

A rating measure $\rho : \mathcal{L}_{[0,1]} \to \mathbb{R}$ generates $\mathcal{I}$ if for some ordered partition $(J_1, \ldots, J_n)$ of $\mathbb{R}$ and $k = 1, \ldots, n$,

$$I_k = \{(L, M) \in \mathcal{X} : \rho(L/M) \in J_k\}.$$

- PD, EL, ...

Characterization theorems

Theorem

Fix a collection of scenarios S. A rating criterion $\mathcal{I}$ satisfies [SC] and [SR] if and only if it is generated by

$$\rho(X) = \int_0^1 h(\mathbb{P}(X > x|S_1), \dots, \mathbb{P}(X > x|S_m))\mathrm{d}x, \ \ X \in \mathcal{L}_{[0,1]},$$

for some increasing function $h : [0,1]^m \to \mathbb{R}$ with $h(\mathbf{0}) = 0$.

Example.

$$\rho(X) = \sum_{j=1}^{m} \int_0^\infty h_j(\mathbb{P}(X > x|S_j))\mathrm{d}x,$$

where h_j is an increasing function on $[0,1]$ with $h_j(0) = 0$.

Characterization theorems

Corollary

A rating criterion $\mathcal{I}$ satisfies [SC] and [LI] if and only if it is generated by

$$\rho(X) = \int_0^1 h(\mathbb{P}(X > x))\mathrm{d}x, \ \ X \in \mathcal{L}_{[0,1]}$$

for some increasing function $h : [0, 1] \to \mathbb{R}$ with $h(0) = 0$.

Progress

Primary examples

Example (the S&P scenario factor).

- S&P chooses scenarios $S_i = \{S = s_i\}$, $i = 1, \dots, m$ to reflect different economic situations
  - e.g. the Great Depression, the Subprime Crisis, ...
  - from the most adverse (S_1) to the safest (S_m)
- "treat" each loss as a function of S, i.e. $L = f_L(S)$
- a bond (L, M) is given a rating $k \in \{1, \dots, m+1\}$ if it can survive scenarios $S_k, \dots, S_m$ but not S_{k-1}, i.e.

$$\mathcal{I}(L, M) = \max\{k \in \{1, \dots, m+1\} : f_L(s_{k-1}) > 0\}.$$

- It is a PD criterion if f_L is an increasing function.

Primary examples

Example (the Moody's scenario factor for synthetic CDOs).

- Use a standard Gaussian copula model for the portfolio backing the synthetic CDO
- Specify three scenarios: $S_i = \{\Sigma = \Sigma_i\}$, $i = 1, 2, 3$ representing low, medium and high correlations in the portfolio
- Specify weights $(\lambda_1, \lambda_2, \lambda_3) = (0.7, 0.2, 0.1)$
- Calculate

$$\rho\left(\frac{L}{M}\right) = \sum_{i=1}^{3} \lambda_i \mathbb{E}\left[\frac{L}{M} \middle| S_i\right]$$

- Give rating according to the above quantity

Examples of rating measures

Choose $h(u_1, \dots, u_m) = \sum_{i=1}^m a_i u_i$, $(u_1, \dots, u_m) \in [0,1]^m$, $a_1, \dots, a_m \geq 0$, one gets

$$\rho(X) = \sum_{i=1}^{m} a_i \mathbb{E}[X|S_i], \quad X \in \mathcal{L}_{[0,1]}.$$

This recovers the Moody's formula by setting $S_1, \dots, S_m$ according to correlations.

Examples of rating measures

For some $a_1, \dots, a_m \geq 0$ with $\sum_{i=1}^m a_i = 1$ and $p \in (0,1)$, let $\mathrm{VaR}_p(X|S_i)$ be the conditional p-quantile of X under S_i.

- Average VaR:

$$\rho(X) = \sum_{i=1}^m a_i \mathrm{VaR}_p(X|S_i), \quad X \in \mathcal{L}_{[0,1]}.$$

- Max VaR:

$$\rho(X) = \bigvee_{i=1}^n \mathrm{VaR}_p(X|S_i), \quad X \in \mathcal{L}_{[0,1]}.$$

Examples of rating measures

For some $a_1, \dots, a_m \geq 0$ with $\sum_{i=1}^m a_i = 1$ and $p \in (0,1)$,

- Average ES:

$$\rho(X) = \sum_{i=1}^m a_i \mathrm{ES}_p(X|S_i), \quad X \in \mathcal{L}_{[0,1]},$$

- Max ES:

$$\rho(X) = \frac{1}{1-p} \int_p^1 \left(\bigvee_{i=1}^n \mathrm{VaR}_q(X|S_i) \right) \mathrm{d}q, \quad X \in \mathcal{L}_{[0,1]}.$$

- Examples in W.-Ziegel'18

Our contributions

Our contributions

- rigorously formulate the phenomenon of tranche exploitation; PD leads to tranche exploitation, whereas EL does not;
- introduce self-consistent rating criteria; EL is self-consistent, whereas PD is not;
- characterize all rating criteria satisfying two axioms of self-consistency and scenario-relevance;
- present a set of new examples for a sensible rating criteria.

Thank you

Thank you for your kind attention

This paper is not yet online; we plan to put it on SSRN in a month.

Comments are welcome.