Topics in Actuarial Science

Robust Risk Aggregation
ACTSC 991, Winter 2020

Instructor:	Ruodu Wang, M3 3122, ext. 31569, wang@uwaterloo.ca
Lectures:	2:30 – 3:50 pm Monday and Wednesdays, M3 3103
Tutorials:	N/A
Office hours:	4:00 – 5:00 pm Monday and Wednesdays, or by appointment. You are welcome to drop by my office at any time, and if I am not occupied I will be happy to answer your questions.
Target audience:	Ph.D. students and Master's students interested in research

Content Description

We study the theory of *robust risk aggregation* and its applications. This course covers various topics in this quickly expanding field. The content is mainly based on personal research experience of the instructor and his collaborators.

Generally, robust risk aggregation refers to the study of quantitative characteristics (e.g., risk measures) of an aggregate position (sum, typically) of several risk factors, where some information of the comprehensive model is not available. A special setting which we will focus on is that the marginal distribution of each individual random variable is known and the joint distribution (copula) is unspecific. Problems under this settings trace back to early contributions of Fréchet, Hoeffiding, Hardy-Littlewood, and Kolmogorov. The modern version of this problem is motivated by (i) the large statistical challenges for high-dimensional models and data sparcity, (ii) the common practice in risk evaluation and regulation, (iii) applications from other fields, such as assembly scheduling, game theory, treatment effect analysis, and robust hypothesis testing, (iv) the purely mathematical curiosity.

A large number of mathematical and statistical questions in this field are still open up to now, and likely retain so for the instructor's generation.

The course should be treated as a MATH course. It is proof based.
Not a data/computation course.
Not a finance/economics course.

For general knowledge on risk management applications, please take other courses (such as 964 which I also teach). The depth of the topics will be at the level of recent research advances. By research here I really mean mathematical research.

References

Many results are from recently published research papers and the materials will not follow a particular book. For the sack of completeness, some reference books are

(i) Rüschendorf, L. (2013). *Mathematical Risk Analysis.* Springer Series in Operations Research and Financial Engineering.

(ii) McNeil, A. J., Frey, R. and Embrechts, P. (2015). *Quantitative Risk Management: Concepts, Techniques, Tools.* Revised Edition. Princeton University Press.

- You are not required to purchase those books.
- I will only touch small parts of the those books that are relevant to the topic of the course.
- Reference papers will be provided in the lectures. I will supply the slides after lectures.

Topics and Tentative Schedule

The schedule and topics are very much tentative. We do not follow strictly to the schedule. as we may spend more time on some details, and may not be able to cover all topics listed there. There are also some more recent research papers that are relevant to the topic, and we may discuss some of them in the lectures as well. The course content has six parts.

1. Introduction to robust risk aggregation
2. Classic results on dependence uncertainty
3. Theory of joint mixability
4. Uncertainty bounds for risk measures
5. Selected applications to other fields
6. Other topics and business

	Lectures	Topics
Part I **Introduction to** **robust risk aggregation**	1-2	Mathematical framework The Fréchet problem and its history VaR, ES and risk measures Relation to other problems
Part II **Classic results on** **dependence uncertainty**	3-6	Copulas Comonotonicity/counter-monotonicity Convex order Simple ES bounds VaR bounds for $n = 2$
Part III **Theory of** **joint mixability**	7-10	Definition and examples Basic properties Necessary conditions Sufficient conditions Uniform and Cauchy distributions
Part IV **Uncertainty bounds** **for risk measures**	11-14	VaR and ES bounds Analytical results Distortion risk measures Asymptotic equivalence Dependence uncertainty spread
Midterm (15)		
Part V **Selected applications** **to other fields**	16-19	Robust hypothesis testing Treatment effect analysis Assembly scheduling Limited resources allocation games
Part VI **Other topics and business**	20-24	(Paper discussions) (Student presentations)

Assignments

There will be one question-solving assignment. Depending on the number of students, we may also have two assignments, or no assignments.

In-class presentation and project

Each student is expected to give one oral presentation on research topics/papers related to the course during the lectures. The typical length of each presentation is about 30 minutes which also depends on the number of students enrolled in the class. The presentation will lead to a small research project (it could be either a problem solving essay, a review essay or an application essay).

Midterm

There will be one midterm on the technical material, which is currently scheduled on the 15th lecture (March 2, Monday of week 8).

Course Evaluation Breakdown

(1) Assignments 20% (if no assignment, then the weight will be uniformly distributed over the other items);

(2) Midterm 30%;

(3) In-class presentation 25%;

(4) Final project 25%.

Policy on Intellectual Property

Students should be aware that this course contains the intellectual property of their instructor, TA, and/or the University of Waterloo. Intellectual property includes items such as:

Lecture content, spoken and written (and any audio/video recording thereof); Lecture handouts, presentations, and other materials prepared for the course (e.g., PowerPoint slides); Questions or solution sets from various types of assessments (e.g., assignments, quizzes, tests, final exams); and Work protected by copyright (e.g., any work authored by the instructor or TA or used by the instructor or TA with permission of the copyright owner). Course materials and the intellectual property contained therein, are used to enhance a student's educational experience. However, sharing this intellectual property without the intellectual property owner's permission is a violation of intellectual property rights. For this reason, it is necessary to ask the instructor, TA and/or the University of Waterloo for permission before uploading and sharing the intellectual property of others online (e.g., to an online repository).

Permission from an instructor, TA or the University is also necessary before sharing the intellectual property of others from completed courses with students taking the same/similar courses in subsequent terms/years. In many cases, instructors might be happy to allow distribution of certain materials. However, doing so without expressed permission is considered a violation of intellectual property rights.

Please alert the instructor if you become aware of intellectual property belonging to others (past or present) circulating, either through the student body or online. The intellectual property rights owner deserves to know (and may have already given their consent).

Relevant University Policies:

Policy 71 - Student Discipline Policy 73 - Intellectual Property Rights