CO481/CS467/PHYS467 Assignment 2

Due February 10, 2019, 3:00am

Instruction: Please submit your solutions to Crowdmark by the due date and time. Take special care to place the answer to each question in the right place.

Question 1. Teleportation of a system entangled with a reference [5 marks]

Robin prepares an *arbitrary* bipartite quantum state $|\psi\rangle_{RM}$, where R and M are d- and 2-dimensional respectively, for some $d \in \mathbb{N}$. Robin gives the system M to Alice. Alice and Bob share the state $|\Phi_0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB}$ on systems A and B, each with 2 dimensions. Show that, for any $|\psi\rangle_{RM}$ (which is unknown to Alice and Bob), if Alice and Bob apply the teleportation protocol to system M, and relabel Bob's system as D, the resulting state in RD is $|\psi\rangle_{RD}$ (up to an overall phase).

Hint 1: use an appropriate way to express $|\psi\rangle_{RM}$ on the composite system R and M.

Hint 2: You can specify an ordering of the tensor components for your work, and you may find that it helps to *change* this ordering mid-analysis.

Question 2. Optimality of superdense coding [5 marks]

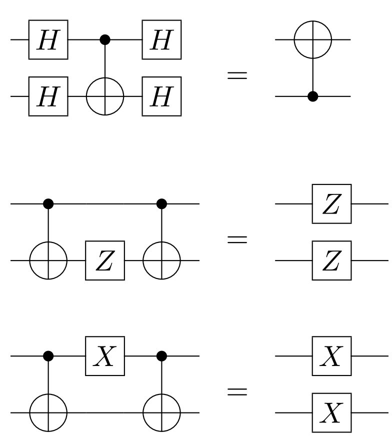
Show that it is impossible for Alice to *communicate* one out of 2^{2n} classical messages to Bob by *sending* him a $\lceil 2^{nr} \rceil$-dimensional quantum system for $0 \leq r < 1$ and a large integer n, even if they are allowed to use an arbitrarily complex entangled state. You can use the principles of no signalling and the no discounted-lunch in the presence of entanglement.

Question 3. Circuit identities [7 marks]

Recall that X and Z stand for the Pauli matrices σ_x and σ_z, and H is the Hadamard gate.

(a) [1 mark] Verify that $HXH = Z$.

(b) [6 marks] Verify the following circuit identities:



Question 4. Universal set of quantum gates [6 marks]

The gates in this question are as defined in class.

For each of the following set of gates, determine if it is universal, and prove your assertion. You may use the fact that $\{\textsc{cnot},\ H, T\}$ is universal.

(a) [2 marks] $\{\textsc{cnot},\ T\}$

(b) [4 marks] $\{\textsc{c}\text{-}Z, K, T\}$, where $\textsc{c}\text{-}Z$ denotes a controlled-Z gate and $K = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$