CO481/CS467/PHYS467 Assignment 2

Due February 10, 2025, 8:30am

Instruction: Please submit your solutions to Crowdmark by the due date and time. Take special care to place the answer to each question in the right place.

Question 1. Teleportation of a system entangled with a reference [5 marks]

Robin prepares an *arbitrary* bipartite quantum state $|\psi\rangle_{RM}$, where R and M are d- and 2-dimensional respectively, for some $d \in \mathbb{N}$. Robin gives the system M to Alice. Alice and Bob share the state $|\Phi_0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB}$ on systems A and B, each with 2 dimensions. Show that, for any $|\psi\rangle_{RM}$ (which is unknown to Alice and Bob), if Alice and Bob apply the teleportation protocol to system M, and relabel Bob's system as D, the resulting state in RD is $|\psi\rangle_{RD}$ (up to an overall phase).

Hint 1: use an appropriate way to express $|\psi\rangle_{RM}$ on the composite system R and M.

Hint 2: You need to pick an ordering of the registers for your work, and *changing* this ordering mid-analysis can give simpler looking equations.

Question 2. Optimality of superdense coding [5 marks]

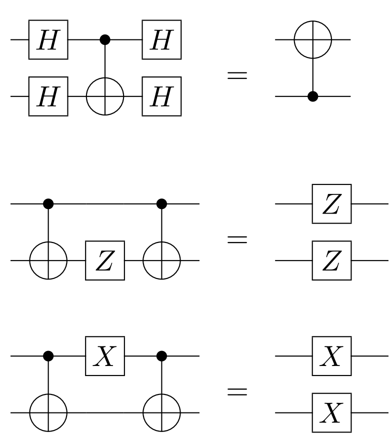
Show that it is impossible for Alice to *communicate* one out of 2^{2n} classical messages to Bob by *sending* him a $\lceil 2^{nr} \rceil$-dimensional quantum system for $0 \leq r < 1$ and a large integer n, even if they are allowed to use an entangled state of arbitrary complexity and dimension. You can use the principles of no signalling and the no discounted-lunch in the presence of entanglement.

Question 3. Circuit identities [7 marks]

Recall that X and Z stand for the Pauli matrices σ_x and σ_z, and H is the Hadamard gate.

(a) [1 mark] Verify that $HXH = Z$.

(b) [6 marks] Verify the following circuit identities:



Question 4. Universal set of quantum gates [6 marks]

The gates in this question are as defined on p34 of topic05-2: H is the Hadamard gate and $T = R_z(\pi/4)$.

For each of the following set of gates, determine if it is universal, and prove your assertion. You may use the fact that $\{\textsc{cnot},\ H, T\}$ is universal.

(a) [2 marks] $\{\textsc{cnot},\ T\}$

(b) [4 marks] $\{\textsc{c}\text{-}Z, K, T\}$, where $\textsc{c}\text{-}Z$ denotes a controlled-Z gate, $Z = \sigma_z$, and $K = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$