Topics 11-13: Quantum cryptography in the presence of noise and adversaries

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    - Encryption
    - Classical one-time pad
    - Key distribution problem
    - QKD through a noiseless insecure channel (BB84, E92, and their relation)
    - QKD through noisy insecure channels

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Additional reading: chapter "Quantum cryptology" by Hoi-Kwong Lo in "Introduction to Quantum Computation and Information" by Lo, Popescu, and Spiller.

Cryptography: information processing to protect honest parties from the action of malicious adversaries.

Quantum money (Wiesner late 60's, published 83)

Question: how to make money that cannot be forged?

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A better tomorrow (1985)

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New: serial number is stored as an n-qubit quantum state on the banknote: $H^{b_1}|c_1\rangle \otimes H^{b_2}|c_2\rangle \otimes \cdots \otimes H^{b_n}|c_n\rangle$

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The bank keeps a record of both c1 c2 ... cn and b1 b2 ... bn.

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To spend the money, the banknote should be verified by the bank who measures in the {|0>,|1>} or the {|+>,|->} basis according to b1 b2 ... bn.

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A forger having the banknote alone cannot learn c1 c2 ... cn, b1 b2 ... bn from the quantum state, nor to clone it (cf A4Q1).

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Conjugate coding:
The bit c is encoded in one of the two conjugate (or mutually unbiased) bases {|0>,|1>} or {|+>,|->}. Measuring in one basis completely destroys the info in the other, so if you have the state but not knowing the basis:
(a) it's hard to learn c
(b) inevitably disturbs the quantum state

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For Wiesner's quantum money scheme, the limited prob of success to learn each bit of the serial number is bootstrapped to suppress the probability to learn the entire serial #.

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(a) Yes, (b) No.

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(a) Yes, (b) No.

Furthermore, attempts to learn about c1 ... cn and b1 ... bn alters the original quantum state (lose the money) and the devious behavior may be caught (risk being put in jail).

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1. Binding: Alice cannot commit to one bit, and reveal a different one.
2. Concealing: Bob cannot learn the committed bit before it's revealed.

e.g., If Alice has a good lockbox, she can "commit" by writing b on a piece of paper, locking it in the box, and giving the box to Bob without the key. She can reveal the bit later by giving Bob the key.

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In this protocol:

1. binding property comes from Alice not having physical access to the paper after the commit phase

2. concealing property comes from assuming that the lockbox cannot be opened without the key (similar to the assumption for the security of cash.)

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Coin tossing provides trusted randomness for secure distributed computation ... (not just online gambling).

Plausible quantum scheme for bit commitment

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   $H^{\otimes n}\left(|c_1\rangle|c_2\rangle\cdots|c_n\rangle\right)$ if b=1

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Reveal phase:

1. Alice sends b and c1 c2 ... cn to Bob.
2. Whether b = 0 or 1, roughly half of the qubits are measured in the "correct" basis. Those outcomes should be consistent with c1 c2 ... cn. Bob accepts only if so.

e.g., n = 10, c1 ... cn = 1100100011

if b=0, state sent to Bob:

$|1\rangle\ |1\rangle\ |0\rangle\ |0\rangle\ |1\rangle\ |0\rangle\ |0\rangle\ |0\rangle\ |1\rangle\ |1\rangle$

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Bob measures in basis:

$$+\ \ +\ \ \times\ \ \times\ \ +\ \ \times\ \ +\ \ +\ \ \times\ \ \times$$

$+ : \{|0\rangle, |1\rangle\}$

$\times : \{|+\rangle, |-\rangle\}$

Question: which can be the 10-bit meas outcome?

(a) 1 0 0 1 1 0 0 0 1 1
(b) 1 1 0 1 1 1 0 0 1 0
(c) 0 1 0 0 1 0 1 0 1 1

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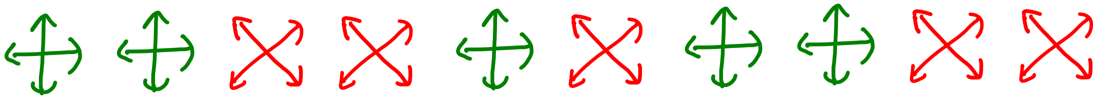

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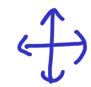
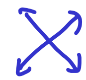
 : $\{|0\rangle, |1\rangle\}$

 : $\{|+\rangle, |-\rangle\}$

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so answer is (b)

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if b=1, state sent to Bob:

$|-\rangle\ |-\rangle\ |+\rangle\ |+\rangle\ |-\rangle\ |+\rangle\ |+\rangle\ |+\rangle\ |-\rangle\ |-\rangle$

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Because $\begin{matrix} \text{wp } \frac{1}{2} : |0\rangle \\ \text{wp } \frac{1}{2} : |1\rangle \end{matrix} \Rightarrow \rho = \frac{I}{2}$ in each qubit for b=0.

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Can we turn the intuition to a rigorous proof ??

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What about dishonest parties?
Described by adversarial models.

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The interesting cases are:

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Concealing property of proposed scheme:

Bob cannot learn b before the reveal phase because his state for both b=0 and b=1 cases is $(I/2)^{\otimes n}$.

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Recall entanglement can create unexpected quantum correlations, e.g., in the GHZ game!

Mayers, Lo, Chau 96

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This is an example of a no-go theorem. It is a very strong statement since it does not only say that certain scheme is insecure. It says no matter how hard researchers try, all such schemes are insecure.!

E.g., What goes wrong with the proposed scheme --
The "analysis" has a hidden assumption, that Alice is sending Bob a pure state based on c and b1 ... bn. She can instead send n qubits that are entangled with some system in her lab, and create correlations that let her cheat !

Mayers, Lo, Chau 96

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But 2 purifications of the same density matrix are related by an isometry between the purifying systems:

$$\therefore \; |\psi_0\rangle = U \otimes I \, |\psi_1\rangle$$

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* Bob's reduced state and outcomes are unchanged by Alice's cheating (no-signalling) but Alice's message in the reveal phase is correlated with Bob's outcome such that Bob always accepts.

E.g. How to cheat in the proposed scheme:

Original:

3. She sends to Bob: $|c_1\rangle|c_2\rangle\cdots|c_n\rangle$ if b=0

$H^{\otimes n}(|c_1\rangle|c_2\rangle\cdots|c_n\rangle)$ if b=1

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A cheating Alice does not commit to any b nor send any of the above states. Instead, she prepares n copies of Bell pairs, and sends one qubit of each pair to Bob, resulting in the joint state:

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_1B_1} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2B_2} \otimes \cdots \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_nB_n}$$

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Bob follows the protocol, measures B1 B2 ... Bn. But since operations on A1 ... An commute with those on B1 ... Bn, we can analyse the situation as if Alice's operations in the reveal phase happen first !

At the reveal phase, Alice picks b.

If b=0, she measures each of A1 A2 ... An along the {|0>,|1>} basis, the random outcomes c1 c2 ... cn & b=0 are sent to Bob. The postmeasurement state on B1 B2 ... Bn is $|c_1\rangle |c_2\rangle \cdots |c_n\rangle$.

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$$\frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right) = \frac{1}{\sqrt{2}}\left(|++\rangle+|--\rangle\right)$$

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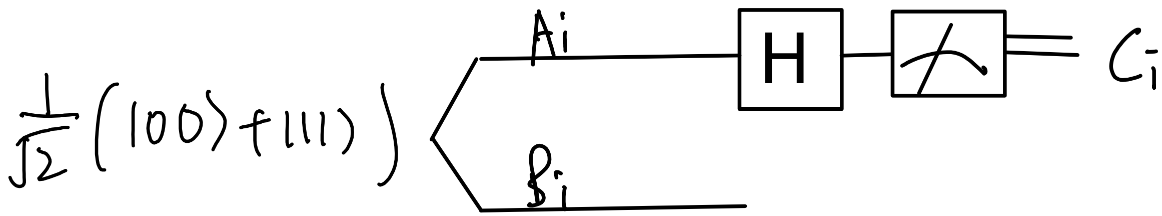
Bob always accepts ....

Alternative derivation of the postmeasurement state:

Measurement along {|+>,|->} =
Hadamard followed by measurement along {|0>,|1>}



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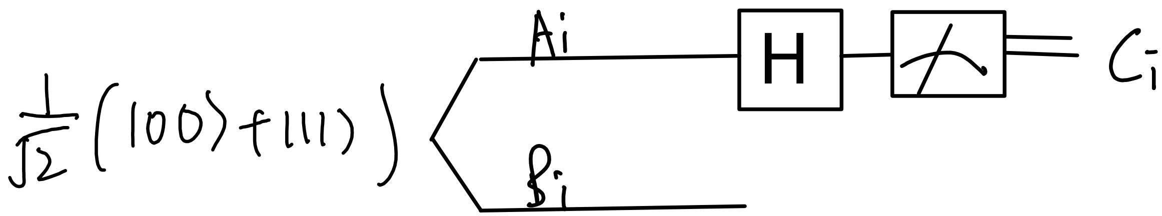
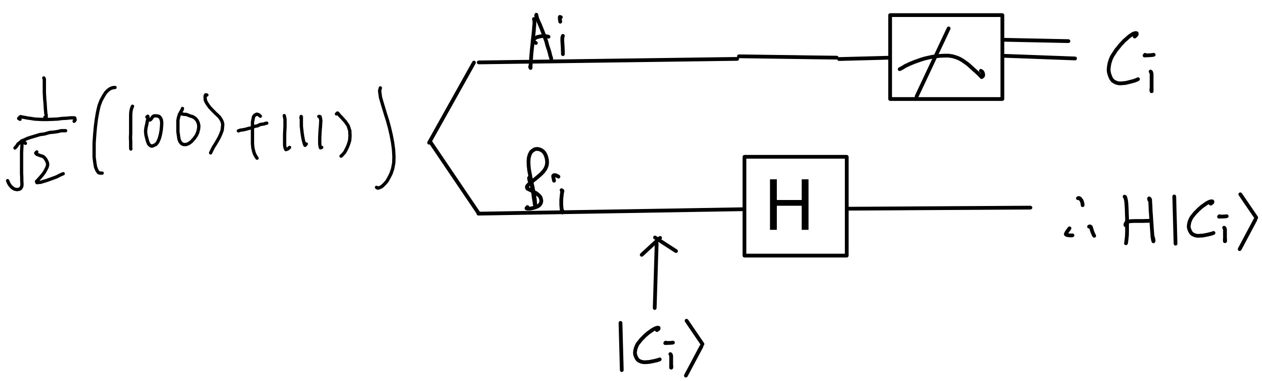


Using the transpose trick (A1), if the transpose of H
is applied to Bi, we get the same state as above:



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Entanglement is really useful, to everyone.

If it helps you teleport, superdense code etc, it can also help the adversary. We must consider the most general attack allowed by QM if there is no reason to restrict the adversary.

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Researchers subsequently add constraints BEYOND QM, e.g., special relativity (that one cannot signal faster than the speed of light) or change the game ...