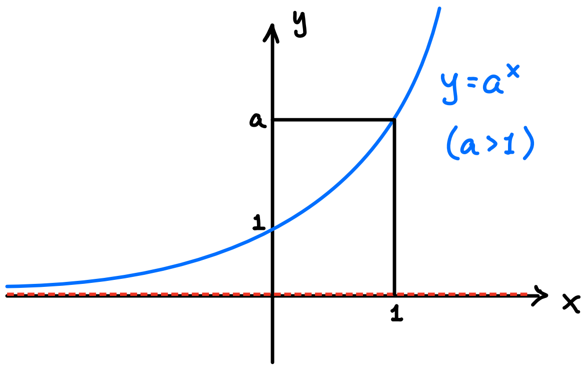
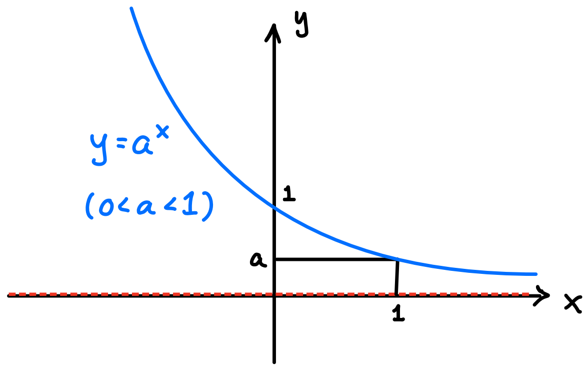
§1.9 - Exponential & Logarithmic Functions

The exponential function $y = a^x$

a = constant
$a > 0, \ a \neq 1$

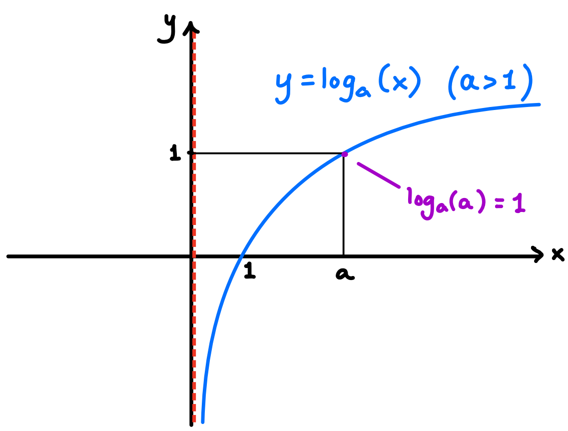


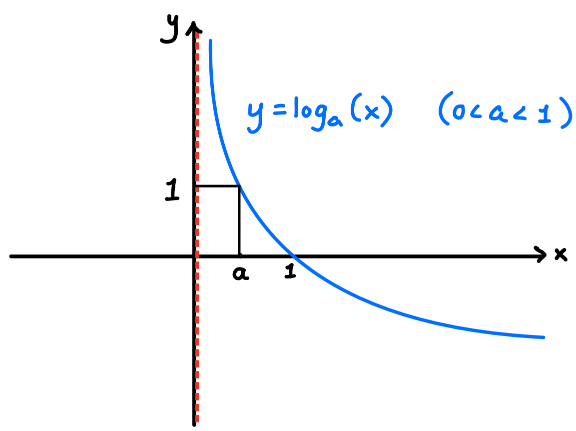


Domain $= (-\infty, \infty)$, Range $= (0, \infty)$

Its inverse is the function $y = \log_a(x)$.

$$\boxed{y = a^x \iff \log_a(y) = x}$$





Domain $= (0, \infty)$, Range $= (-\infty, \infty)$

Ex: $\log_{10}(1000) = 3$ since $10^3 = 1000$.

$\log_2(1/4) = -2$ since $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$.

Exponent Laws	Log Laws
$a^{b+c} = a^b a^c$	$\log_a(bc) = \log_a(b) + \log_a(c)$
$a^{b-c} = \frac{a^b}{a^c}$	$\log_a\left(\frac{b}{c}\right) = \log_a(b) - \log_a(c)$
$(a^b)^c = a^{bc}$	$\log_a(b^c) = c \cdot \log_a(b)$

Ex: Solve the following.

(a) $\log_6(3x) = 2$

Solution: $\log_6(3x) = 2 \Rightarrow 6^2 = 3x$

$\Rightarrow x = 36/3 = \boxed{12}$

(b) $\log_7(x) + \log_7(x+6) = 1$

Solution: $\log_7(x) + \log_7(x+6) = 1$

$\Rightarrow \log_7\big(x(x+6)\big) = 1$

$\Rightarrow 7^1 = x(x+6)$

$\Rightarrow x^2 + 6x - 7 = 0$

$\Rightarrow (x+7)(x-1) = 0.$

$\Rightarrow x = -7 \quad \text{or} \quad x = 1$

[$\log_7(x)$ & $\log_7(x+6)$ are not defined when $x = -7$, but $x = 1$ is okay!]

$\therefore x = 1$ is the only solution

Since a^x and $\log_a(x)$ are inverse, it follows that

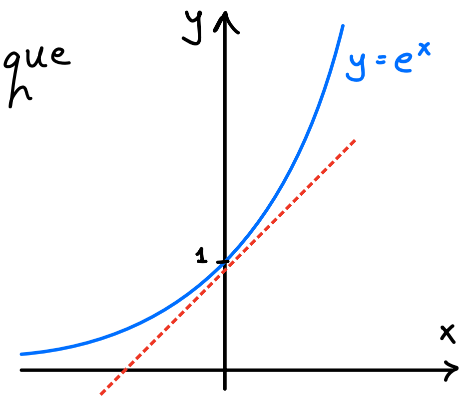
$$\boxed{a^{\log_a(x)} = x \quad , \quad \log_a(a^x) = x}$$

EX: Simplify $5^{3\log_5(x)}$.

Solution: $5^{3\log_5(x)} = 5^{\log_5(x^3)} = \boxed{x^3}$

Special Base: $e \approx 2.71828\ldots$ (Euler's Constant)

We define $a = e$ to be the unique number such that the graph of $y = a^x$ has slope 1 at $x = 0$.

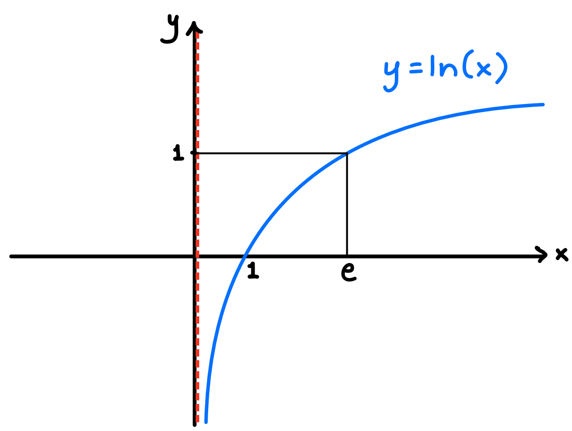


Alternatively, e can be defined by the following famous limit:

$$\boxed{e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n}$$

We call $\log_e(x)$ the <u>natural logarithm</u> and write

$$\boxed{\ln(x) = \log_e(x)}$$



<u>Ex:</u> Solve $\ln(1 + \ln(x)) = 3$

<u>Solution</u>: $\ln(1+\ln(x)) = 3 \quad \Rightarrow \quad e^{\ln(1+\ln(x))} = e^{3}$

$$\Rightarrow \quad 1+\ln(x) = e^{3}$$

$$\Rightarrow \quad \ln(x) = e^{3} - 1$$

$$\Rightarrow \quad e^{\ln(x)} = e^{(e^{3}-1)}$$

$$\Rightarrow \quad \boxed{x = e^{(e^{3}-1)}}$$