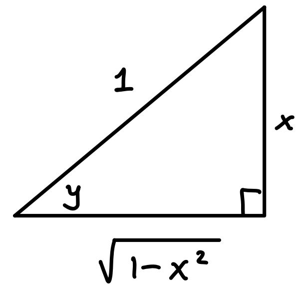
Derivatives of Inverse Trig Functions

What is the derivative of $y = \arcsin(x)$?

Well... $y = \arcsin(x) \Rightarrow \underbrace{\sin y = x}_{\text{differentiate!}} = \frac{x}{1}$



$$\Rightarrow \cos(y) \cdot y' = 1$$

$$\Rightarrow y' = \frac{1}{\cos(y)} = \frac{1}{adj/hyp} = \frac{1}{\sqrt{1-x^2}}$$

Therefore,

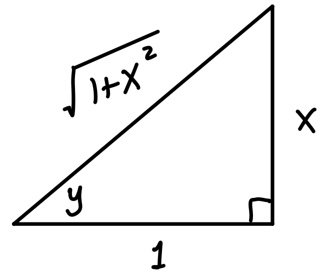
$$\boxed{(\arcsin(x))' = \frac{1}{\sqrt{1-x^2}}.}$$

We can perform a similar calculation to find derivatives of other inverse trig functions!

Ex: Find y' if $y = \arctan(x)$.

Solution: $y = \arctan(x)$

$$\Rightarrow \underbrace{\tan(y) = x}_{\text{differentiate!}} = \frac{x}{1}$$



$$\Rightarrow \quad \sec^2(y) \cdot y' = 1$$

$$\Rightarrow \quad y' = \frac{1}{\sec^2 y} = \cos^2(y) = \left(\frac{\text{adj}}{\text{hyp}}\right)^2 = \underline{\frac{1}{1+x^2}}$$

Therefore,

$$\boxed{\left(\arctan(x)\right)' = \frac{1}{1+x^2}}$$

Ex: Find $f'(x)$ if $f(x) = \arctan(x^3)$.

Solution: By the chain rule,

$$f'(x) = \frac{1}{1+(x^3)^2} \cdot (x^3)' = \boxed{\frac{3x^2}{1+x^6}}$$

Summary:

$$\left(\arcsin(x)\right)' = \frac{1}{\sqrt{1-x^2}}$$

$$\left(\arccos(x)\right)' = \frac{-1}{\sqrt{1-x^2}} \qquad \text{(Exercise!)}$$

$$\left(\arctan(x)\right)' = \frac{1}{1+x^2}$$