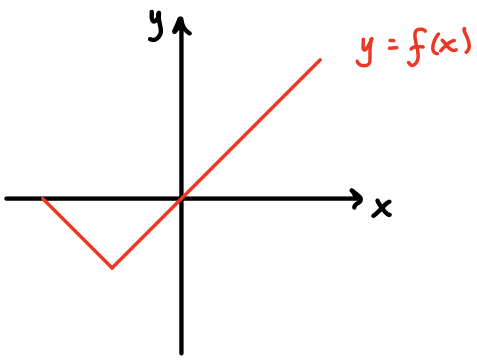
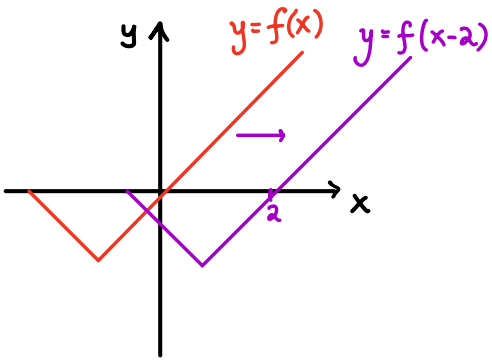
Graphing Functions Using Transformations

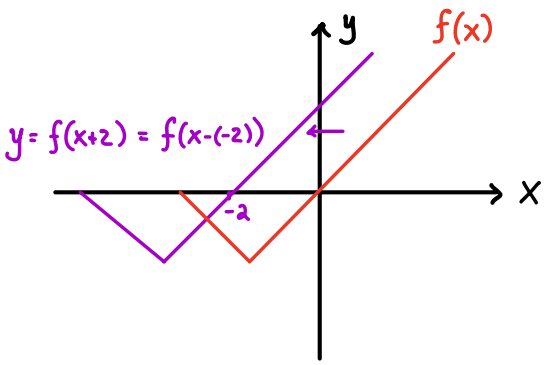
Starting with the graph of $y = f(x)$, we can apply transformations to graph many related functions!



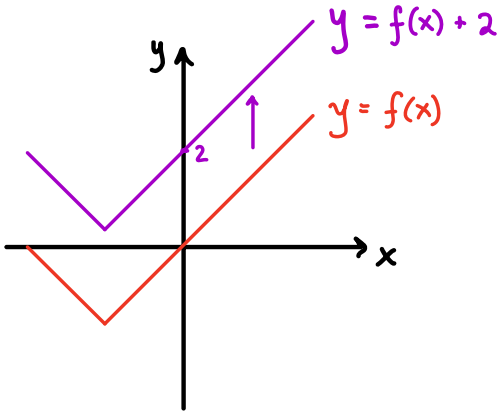
$y = f(x-a)$ $\Rightarrow$ Shift by a units in the positive x-direction

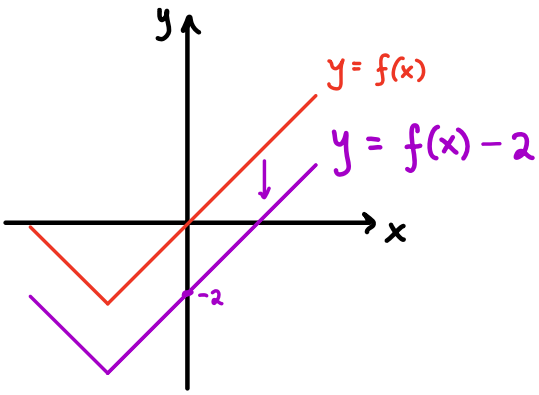
Replace x with x-a, a = constant





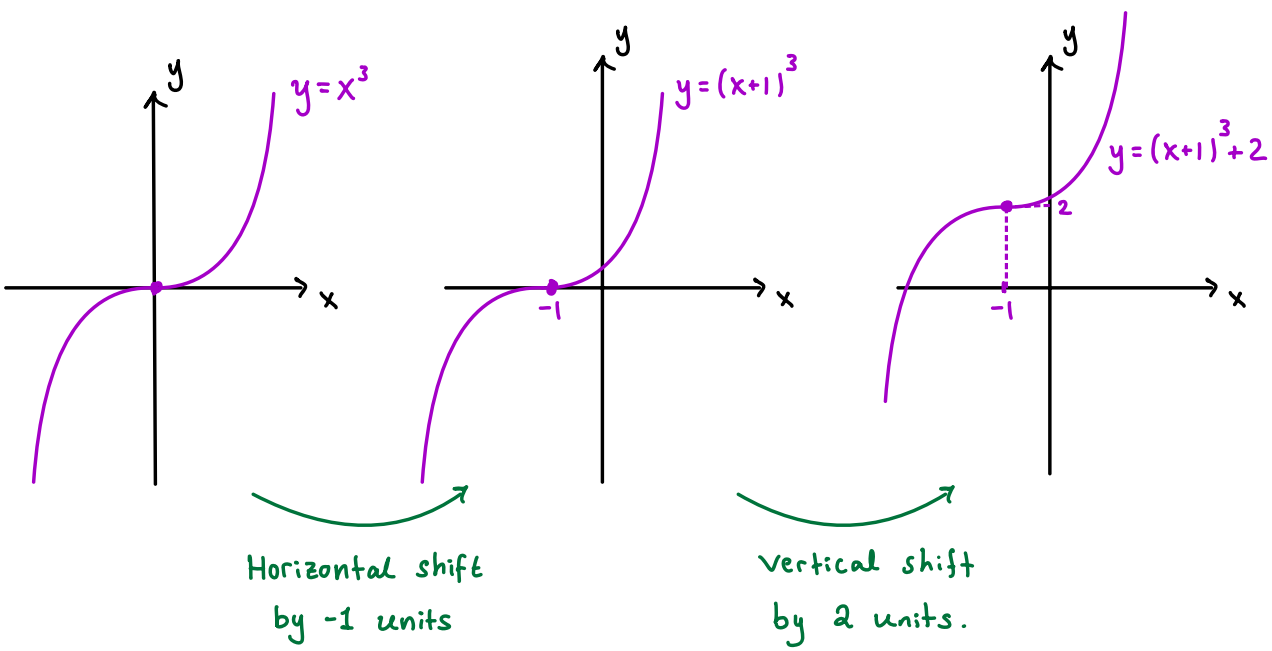
$y = f(x) + a$ (or $y - a = f(x)$) $\Rightarrow$ Shift by a units in the positive y-direction





Ex: Starting with the graph of $y = x^3$, sketch the graph of $y = (x+1)^3 + 2$

Solution

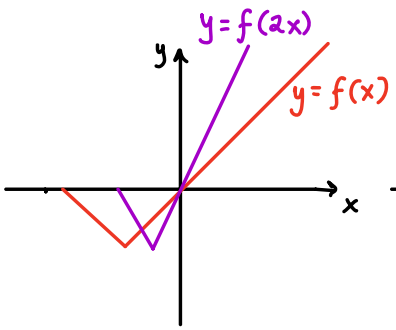


We can also stretch, compress or reflect a

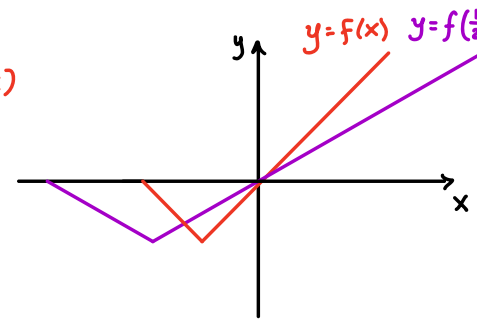
graph about an axis.

$y = f(kx) \Rightarrow$ Horizontal stretch / compression (& reflection over y-axis if $k<0$)

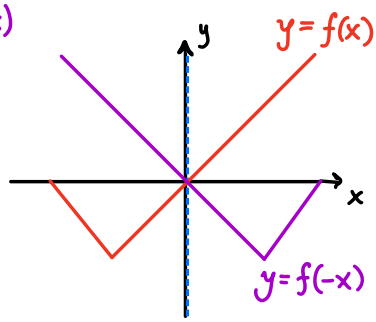
Replace x with kx, k = constant



(compression if $|k|>1$)

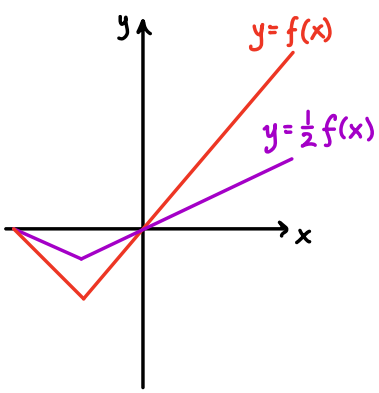


(stretch if $|k|<1$)

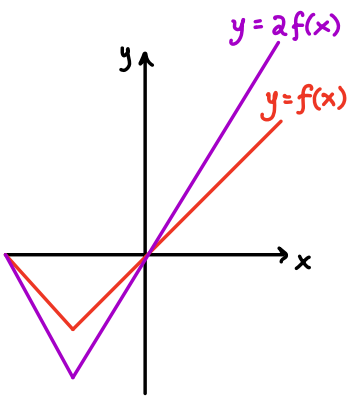


(reflection)

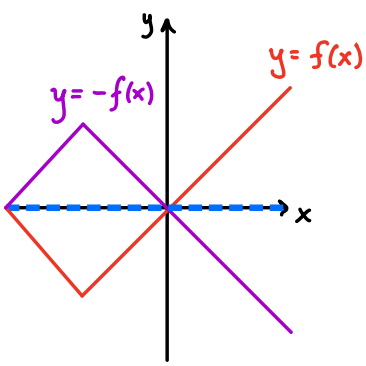
$y = k \cdot f(x) \Rightarrow$ Vertical stretch / compression (& reflection over x-axis if $k<0$)



(compression if $|k|<1$)



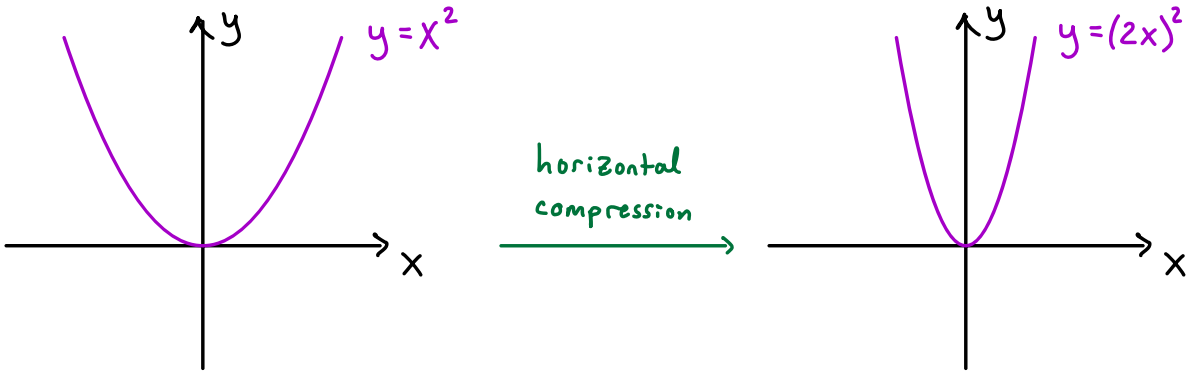
(stretch if $|k|>1$)

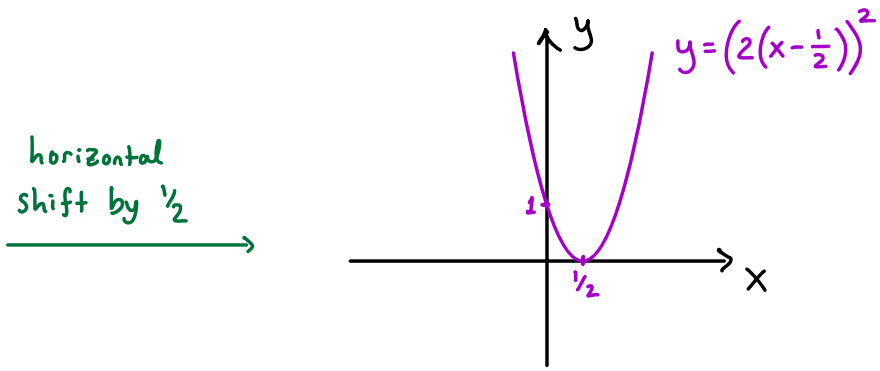


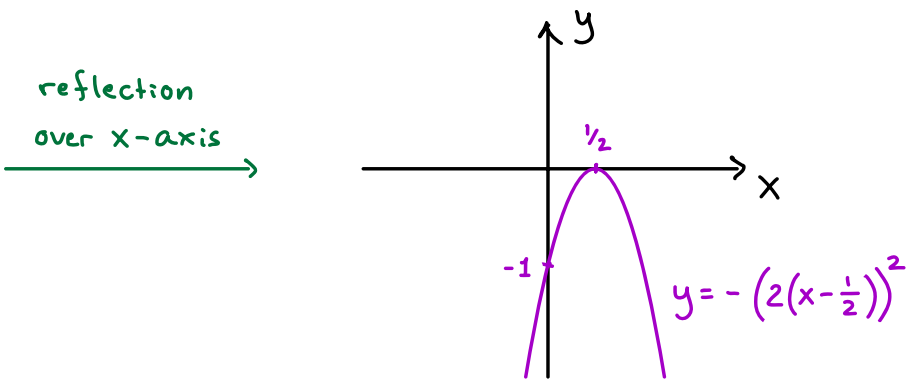
(reflection)

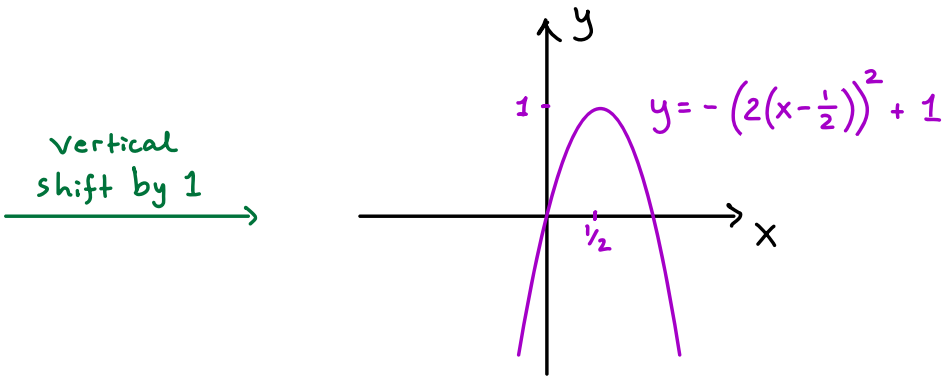
Ex: Sketch the graph of $y = 1 - (2x-1)^2$

Solution: Let's rewrite as $y = -\left(2\left(x-\frac{1}{2}\right)\right)^2 + 1$ to better see the transformations.





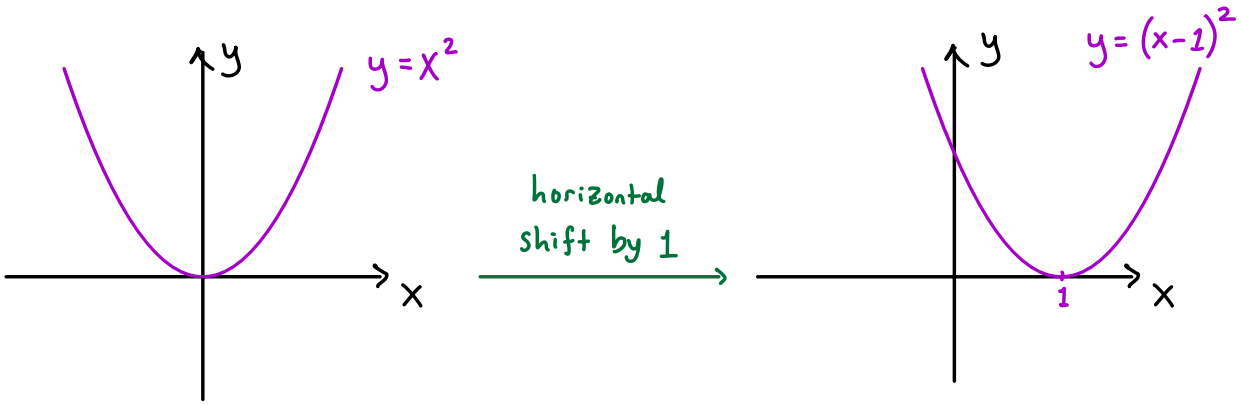


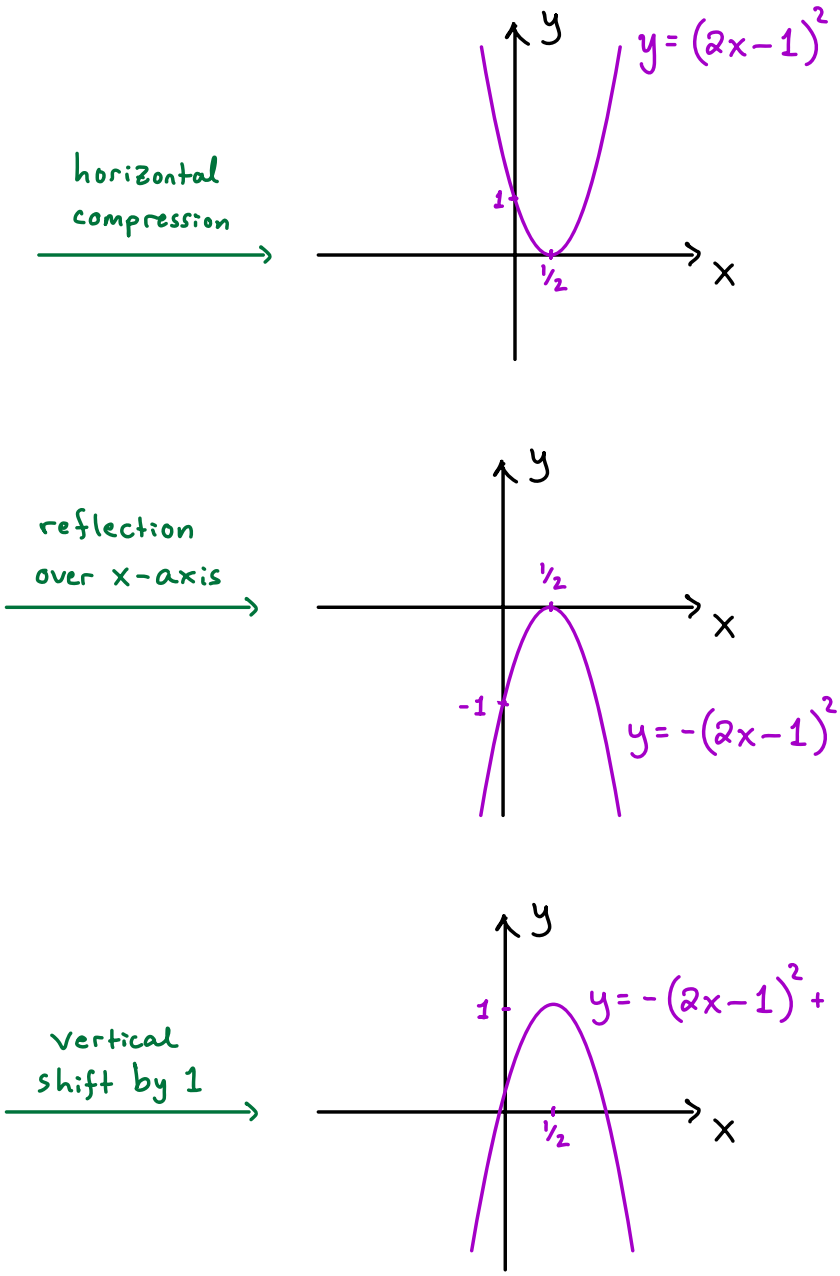


Remark: Be mindful of your order of operations!

The transformations can be applied in various orders, but some adjustments may be required.

In the example above, for instance, we could have instead done the following:

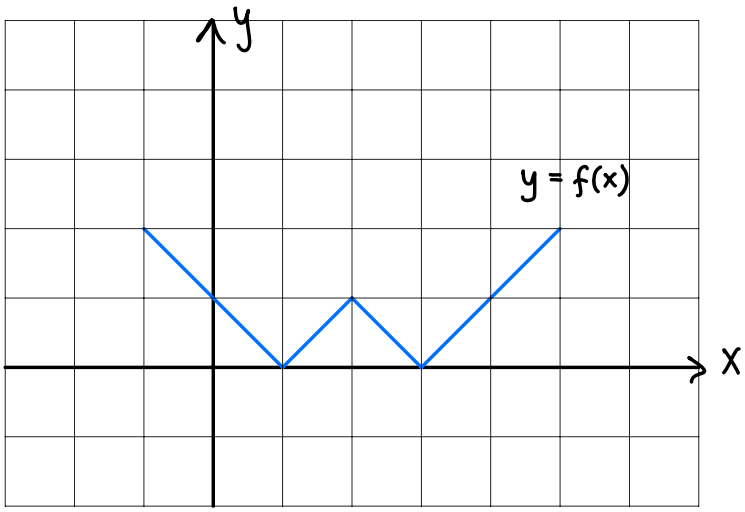




We get the correct equation, $y=-(2x-1)^2+1$, whether we compress <u>then</u> shift or shift <u>then</u> compress, but changing the order requires us to adjust the size of the shift.

Additional Exercise:

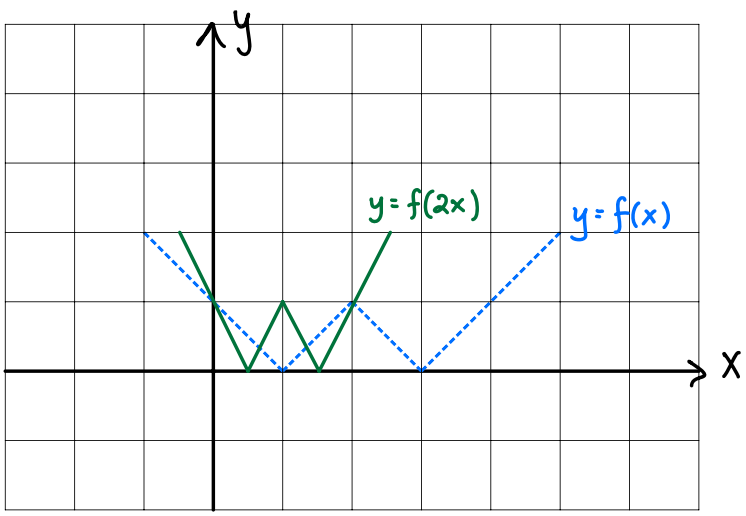
The graph of $y = f(x)$ is shown below.



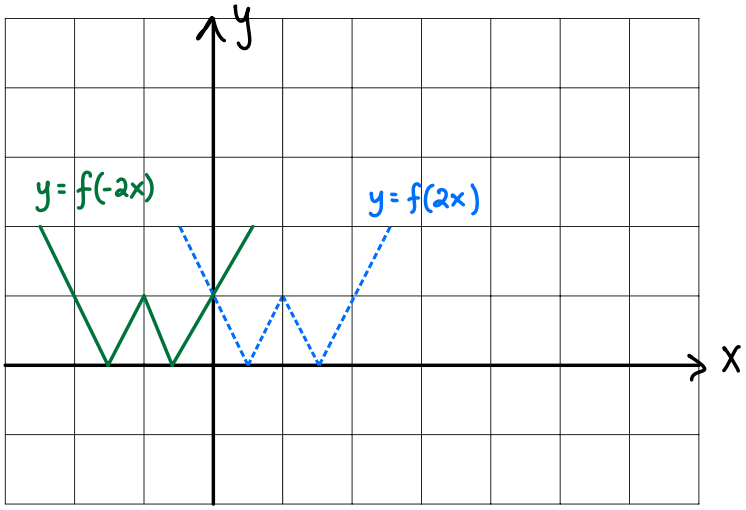
Sketch the graph of $y = 2f(-2x) - 1$

Solution: We'll apply the transformations one by one.

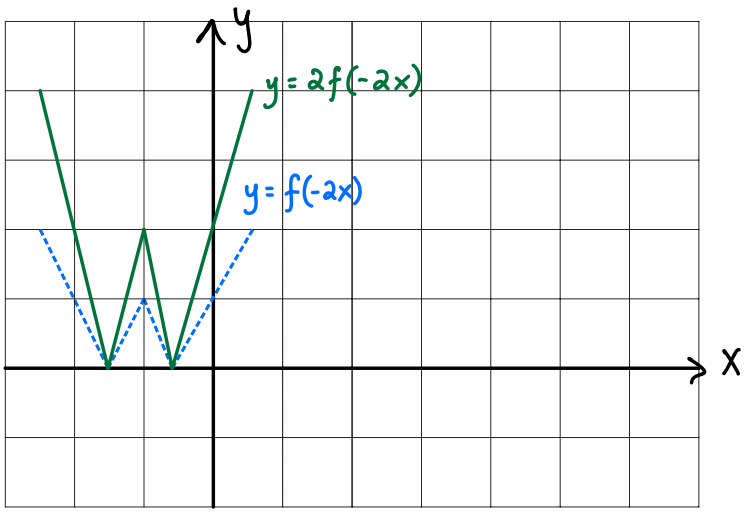
horizontal compression →



reflection over y-axis →



vertical stretch →



Vertical shift →