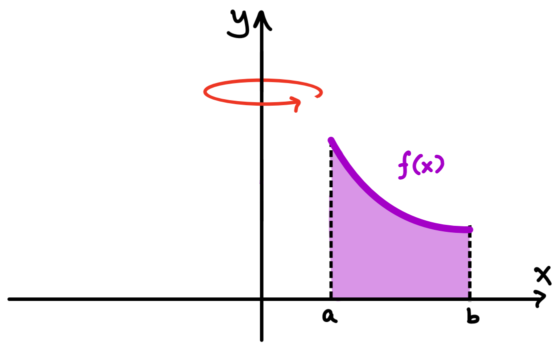
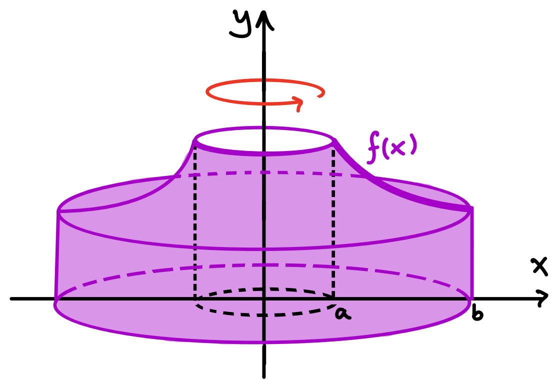
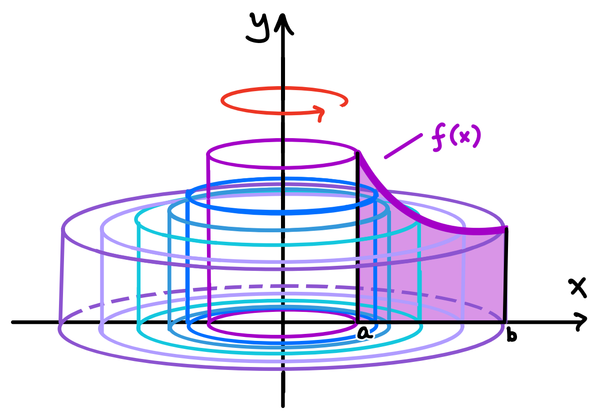
§ 7.2 (continued) – Volumes by Cylindrical Shells

Suppose we wish to compute the volume of the solid of revolution below.





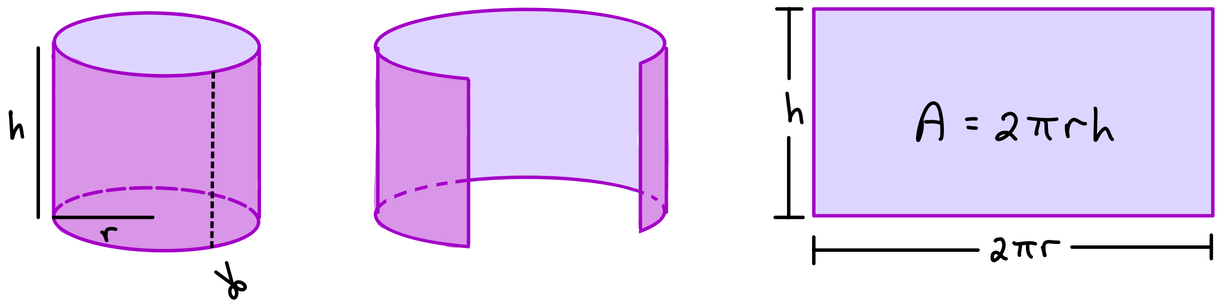
This time, we'll split up the solid into very thin cylindrical shells.



We again have

$$\boxed{\text{Volume} = \int_a^b A(x)\,dx}$$

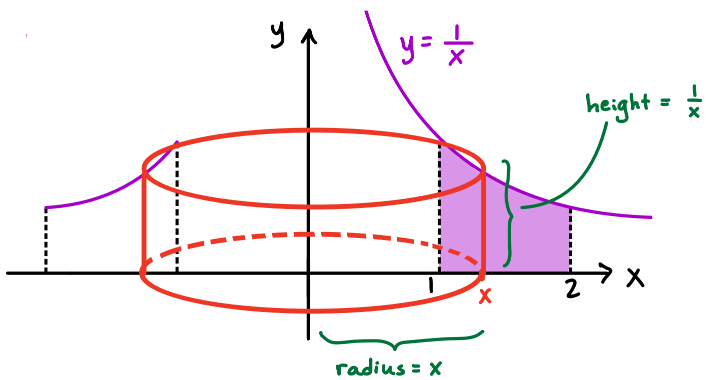
but now $A(x)$ is the surface area of a typical cylindrical shell.



$$\boxed{A(x) = 2\pi \cdot r \cdot h}$$

Ex: Let R denote the region between $y=0$ and $y=\frac{1}{x}$ from $x=1$ to $x=2$. Find the volume of the solid obtained by rotating R about the y-axis.

Solution:

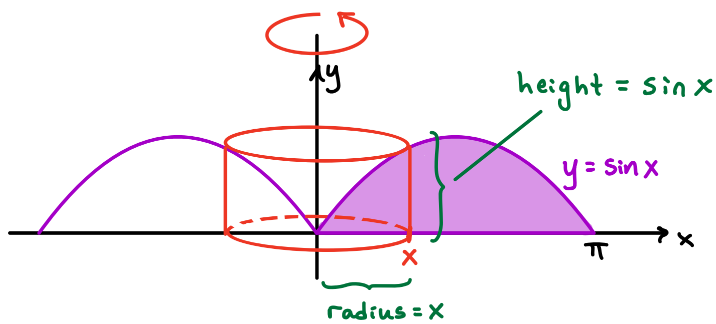


$$\text{Volume} = \int_1^2 A(x)\,dx = \int_1^2 2\pi r h\,dx$$

$$= \int_1^2 2\pi \cdot x \cdot \frac{1}{x}\,dx = \int_1^2 2\pi\,dx = \boxed{2\pi}$$

<u>Ex:</u> Consider the region between $y = \sin x$ and the x-axis for $x \in [0, \pi]$. Set up the integral for the volume of the solid obtained by revolving this region about the y-axis.

<u>Solution</u>:



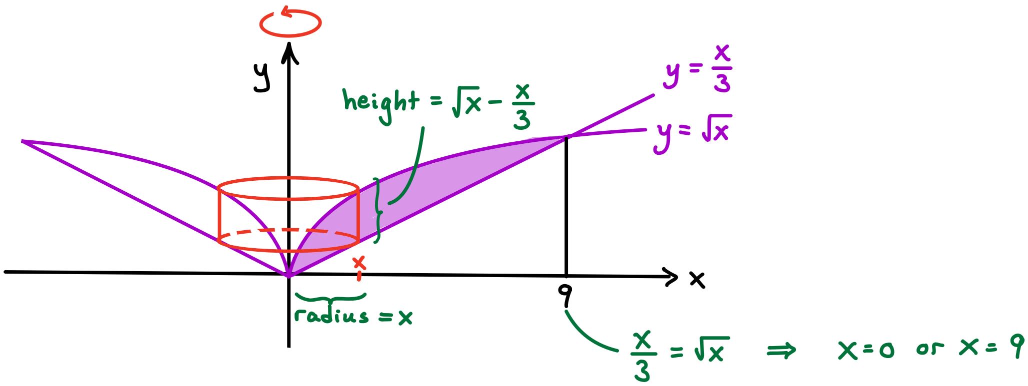
$$\text{Volume} = \int_0^\pi A(x)\,dx = \int_0^\pi 2\pi r h\,dx = \boxed{\int_0^\pi 2\pi x \cdot \sin x\,dx}$$

Ex: Consider the region between $y=\sqrt{x}$ and $y=\frac{x}{3}$.

Set up the integral for the volume of the solid obtained by revolving the region about the given axis.

(a) y-axis

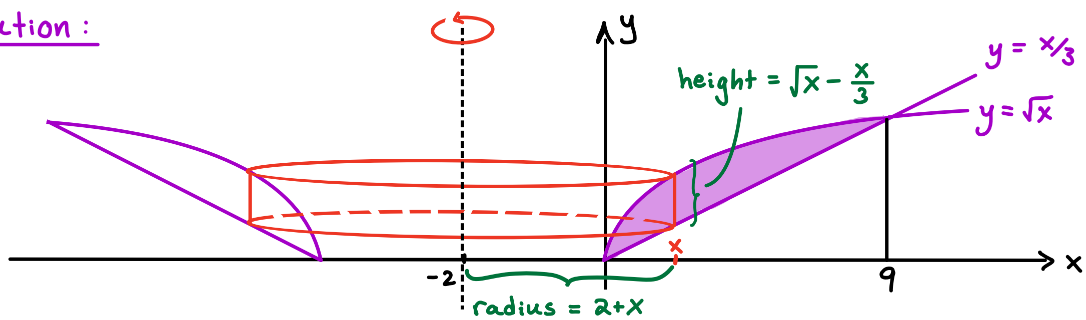
Solution:



$$V = \int_0^9 A(x)\,dx = \int_0^9 2\pi r h\,dx = \boxed{\int_0^9 2\pi x\left(\sqrt{x} - \frac{x}{3}\right)dx}$$

(b) $x=-2$

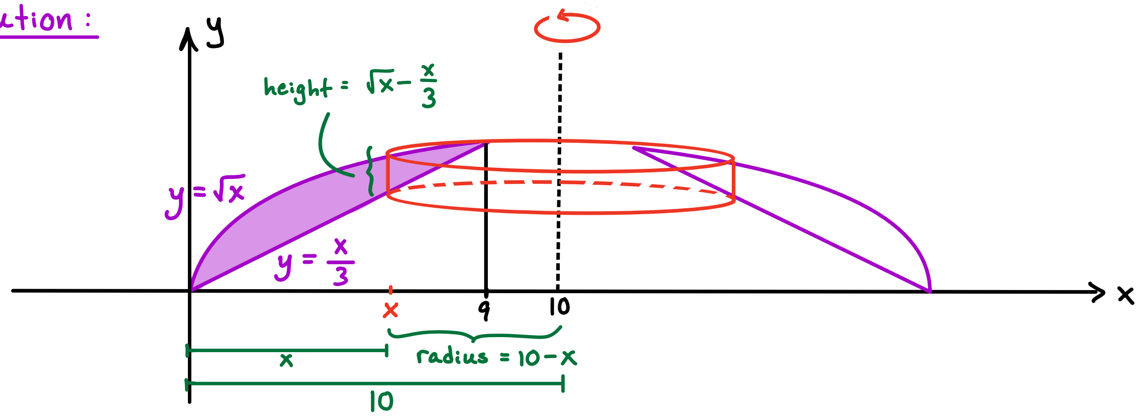
Solution:



$$V = \int_0^9 A(x)\,dx = \int_0^9 2\pi r h\,dx = \boxed{\int_0^9 2\pi(2+x)\left(\sqrt{x} - \frac{x}{3}\right)dx}$$

(c) $x = 10$

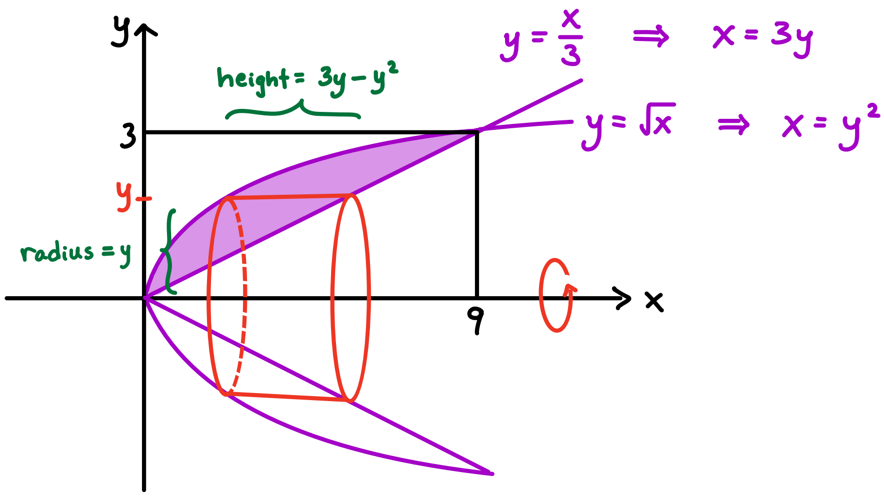
Solution:



$$V = \int_0^9 A(x)\,dx = \int_0^9 2\pi r h\,dx = \boxed{\int_0^9 2\pi(10-x)\left(\sqrt{x} - \frac{x}{3}\right)dx}$$

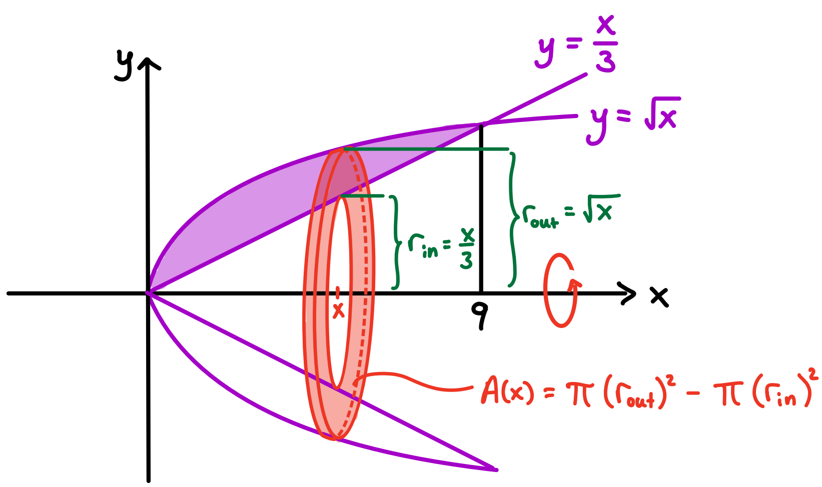
(d) x-axis

Solution: We get a cylindrical shell for each $y \in [0,3]$, so we'll be integrating with respect to y.



$$V = \int_0^3 A(y)\,dy = \int_0^3 2\pi r h\,dy = \boxed{\int_0^3 2\pi y(3y - y^2)\,dy}$$

An easier approach here would be to use washers!



$$V = \int_0^9 A(x)\,dx = \boxed{\int_0^9 \left(\pi(\sqrt{x})^2 - \pi\left(\tfrac{x}{3}\right)^2\right) dx}$$

Summary for cylindrical shells

Revolving around vertical axis? Use functions of x.

Revolving around horizontal axis? Use functions of y.