Mach cones in dusty plasmas: analytical models vs. computer simulation

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OUTLINE

- Introduction to Dusty Plasma Physics
- Structures and Waves in Dust Layers
- Mach Cones in Dust Layers
  - Excited by: moving laser & external particle
  - Experiment, analytical models & simulation
- Polarization Forces on External Particle
- Details of Modeling and Simulation

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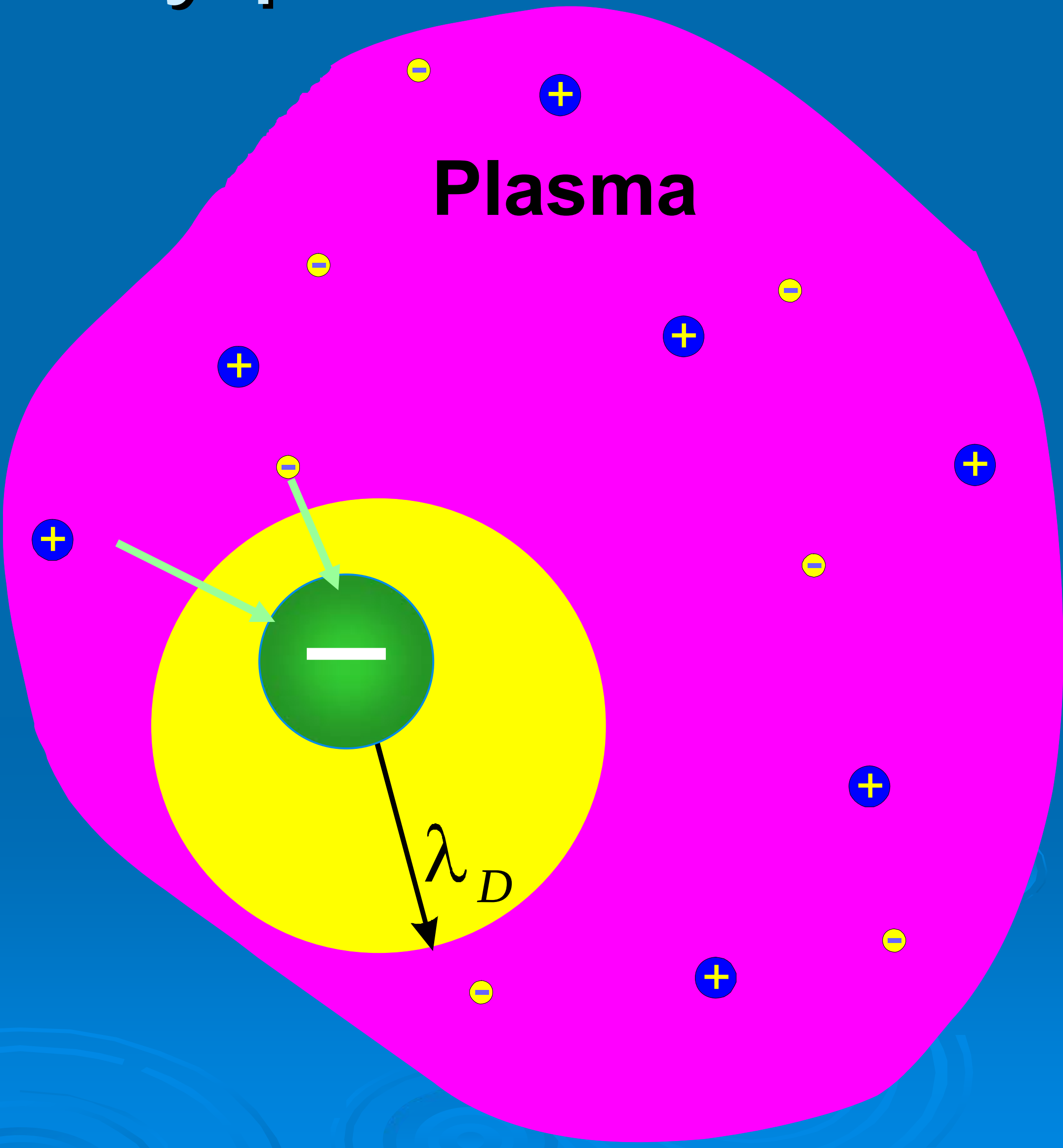
What is a dusty plasma?

electrons + ions + neutrals

+ small particle of solid matter

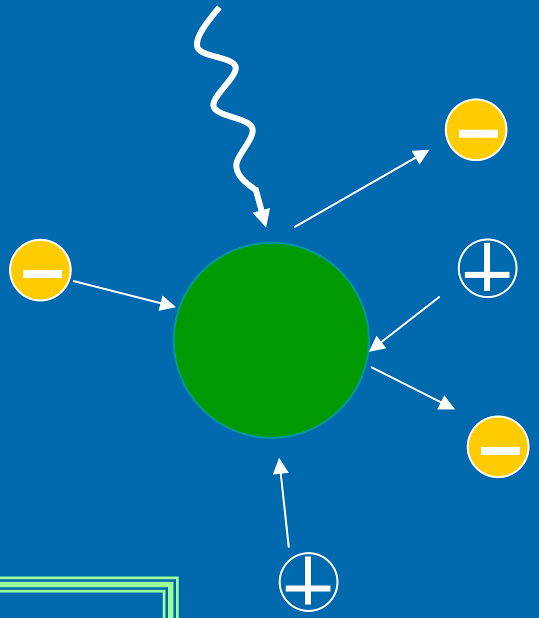
- absorbs electrons and ions
- becomes negatively charged
- Debye shielding



Credit: Merlino

Dust Charging Processes

- electron and ion collection
- secondary emission
- UV induced photoelectron emission



Total current to a grain = 0

$$\Sigma I = I_e + I_i + I_{sec} + I_{pe} = 0$$

Credit: Merlino

The Charge on a Dust Grain

In typical lab plasmas $I_{sec} = I_{pe} = 0$

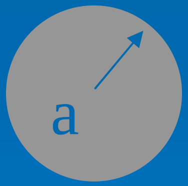
Electron thermal speed >> ion thermal speed so the grains charge to a negative potential V_S relative to the plasma, until the condition $I_e = I_i$ is achieved.

electron repulsion

$$I_e = en_e\sqrt{\frac{kT_e}{m_e}}\;\exp\left(\frac{eV_S}{kT_e}\right)\;\pi a^2$$

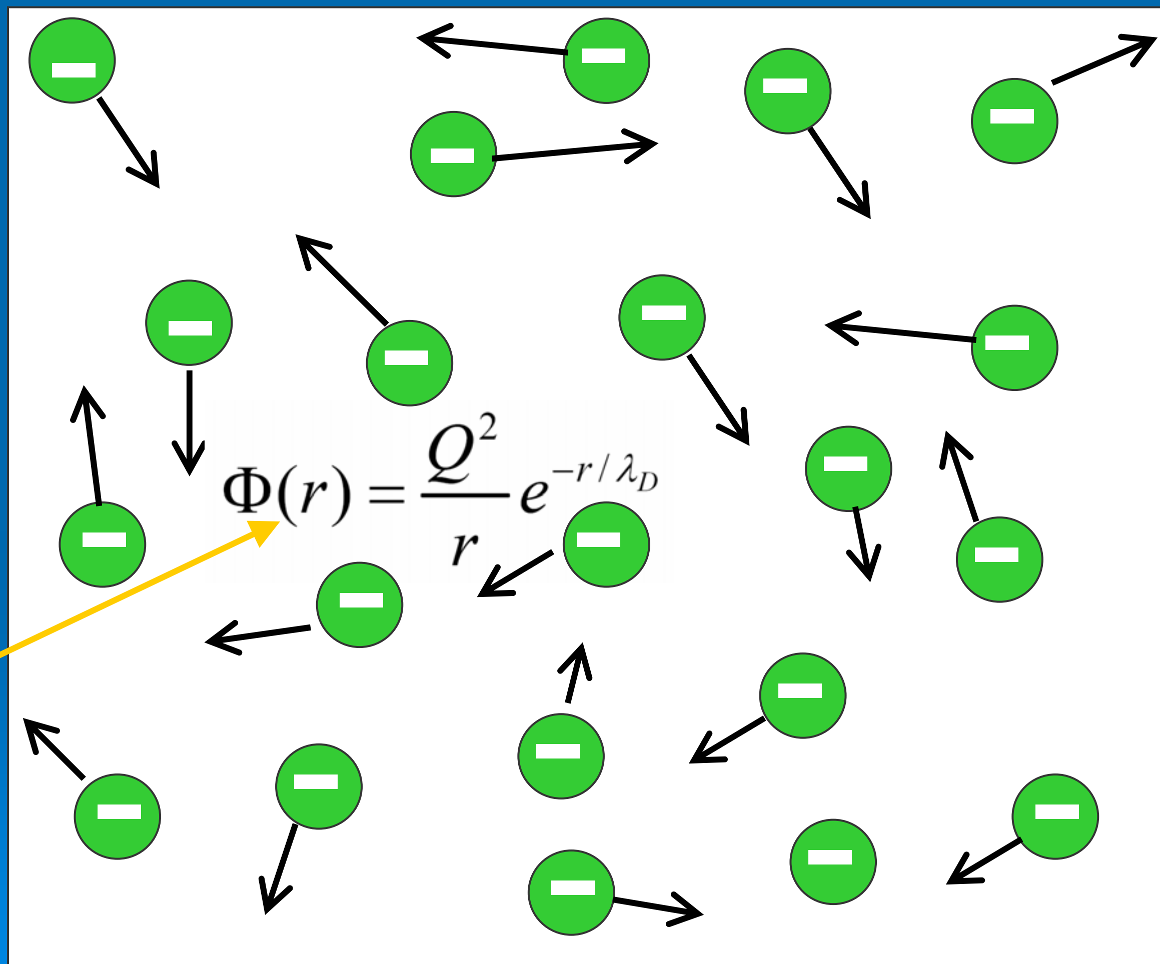
$$I_i = en_i\sqrt{\frac{kT_i}{m_i}}\;\left(1-\frac{eV_S}{kT_i}\right)\;\pi a^2$$

ion enhancement



$Q = (4\pi\varepsilon_o a)\, V_S$

Assembly of highly charged dust particles immersed in a partially ionized plasma



$$\Phi(r) = \frac{Q^2}{r} e^{-r/\lambda_D}$$

Yukawa potential

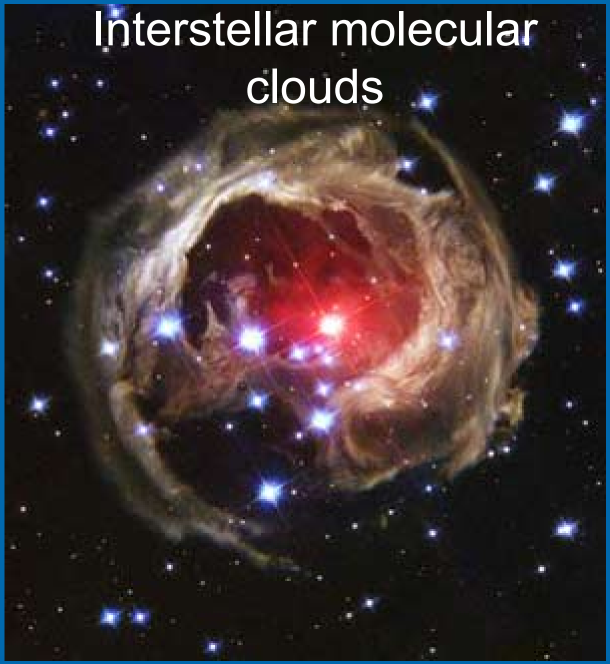
Where are Dusty Plasmas?

- In Nature

- In man-made facilities

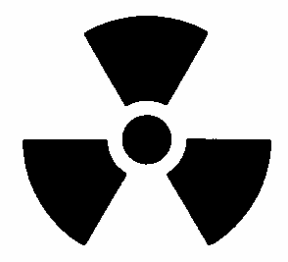
- In research laboratory

Examples of Dusty Plasmas in Nature



Safety issues for fusion !

Radiological



Dust:

- activated
- retains tritium
- ITER safety limit: 350 kg Tungsten dust

Fire & chemical explosion

Hydrogen:

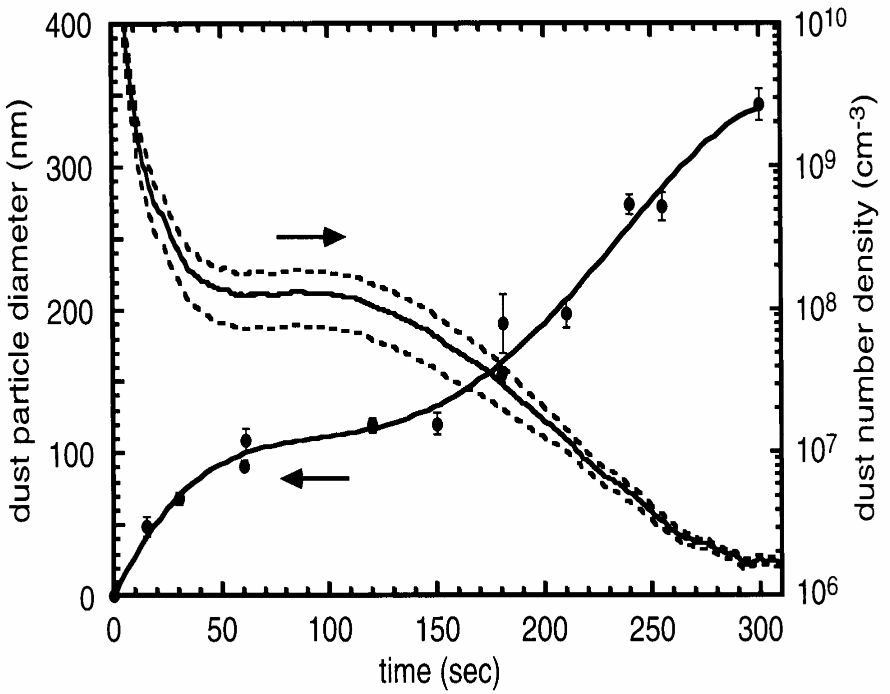
- stored in dust
- released during accidental exposure to:
  - air
  - steam
- ITER safety limit: 6 kg dust allowed on hot surfaces

Phil Sharpe
Fusion Safety Program, Idaho National Laboratory

Dust in Fusion Plasmas Workshop
2005

Credit: Goree

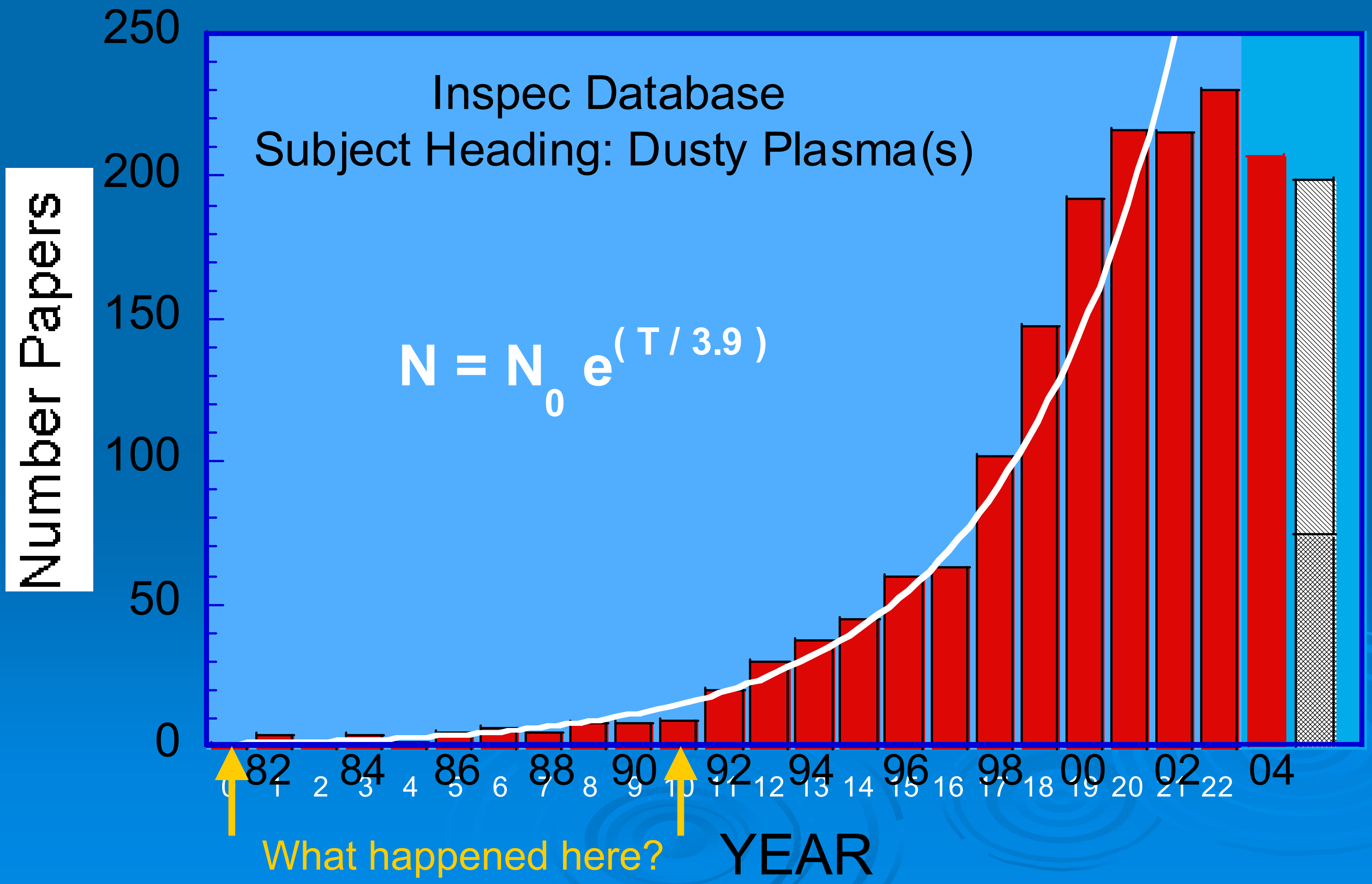
Semiconductor Manufacturing



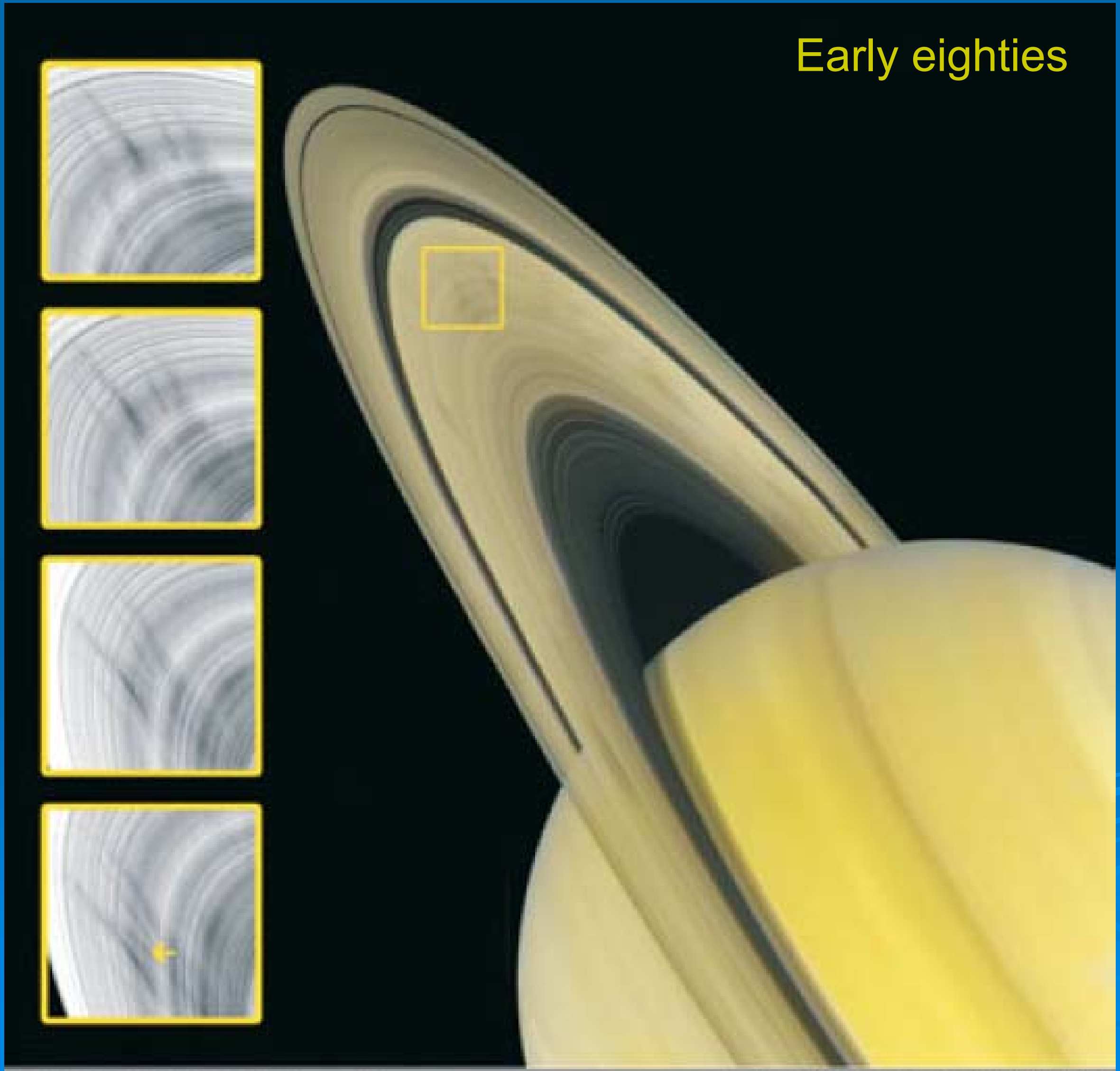
silane (SiH_4) + Ar + O_2 $\rightarrow$ SiO_2 particles

Credit: Merlino

Number of publications on Dusty Plasma



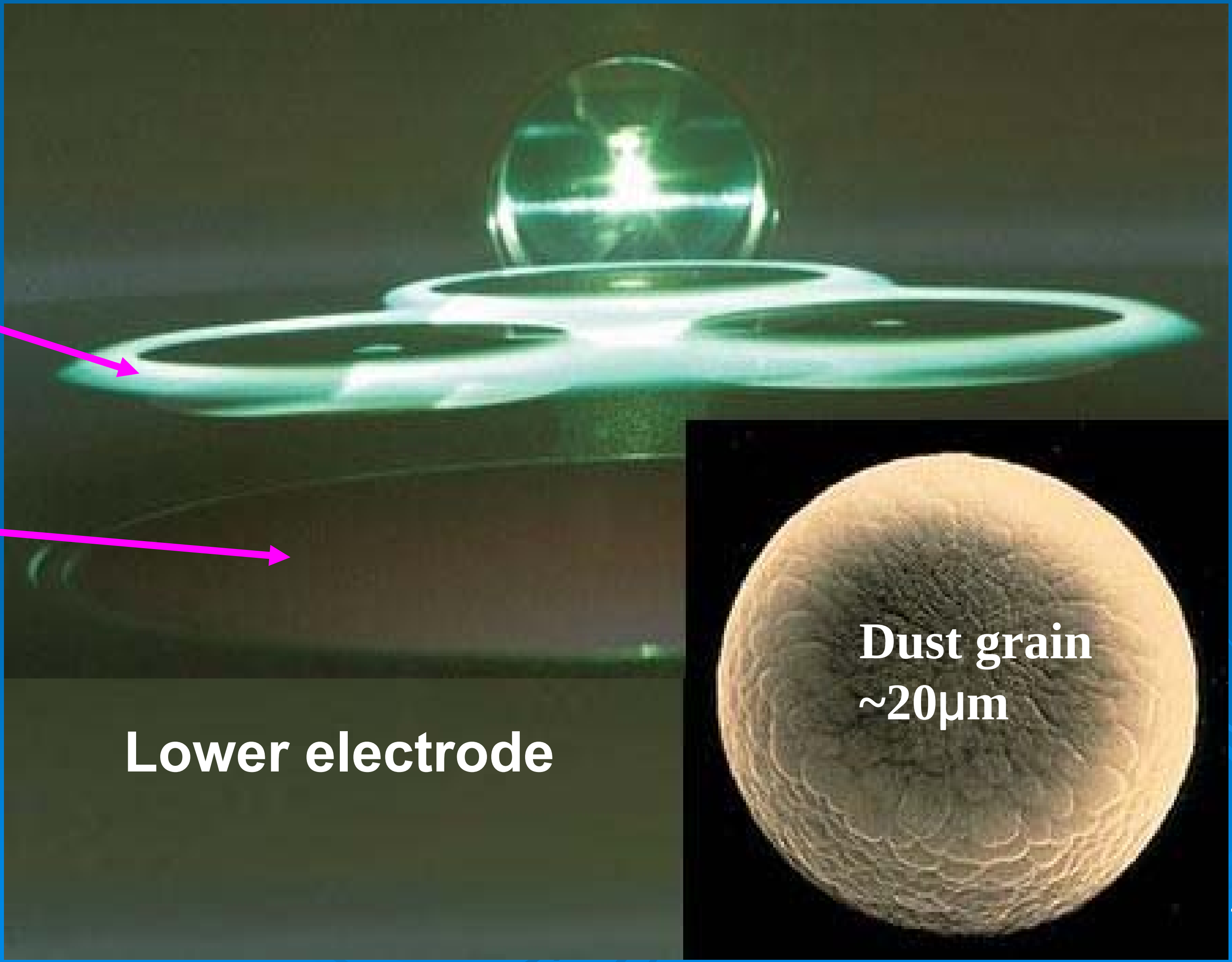
Voyager's images of radial spokes rotating around Saturn's B ring



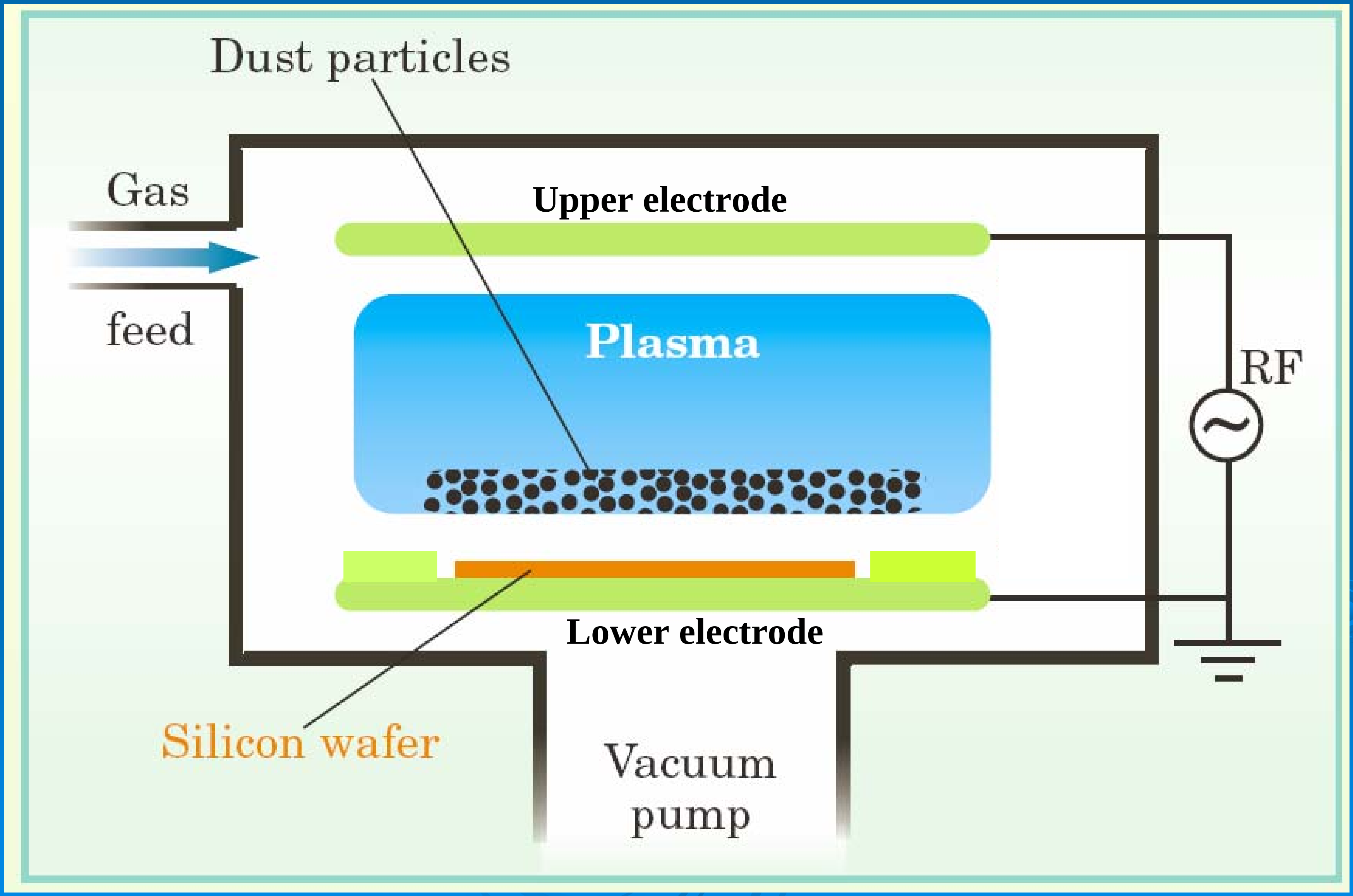
Discovery by Selwyn at IBM in 1989 during plasma etching of Si wafer

Dust cloud

Silicon wafer



Schematics of Selwyn's experiment



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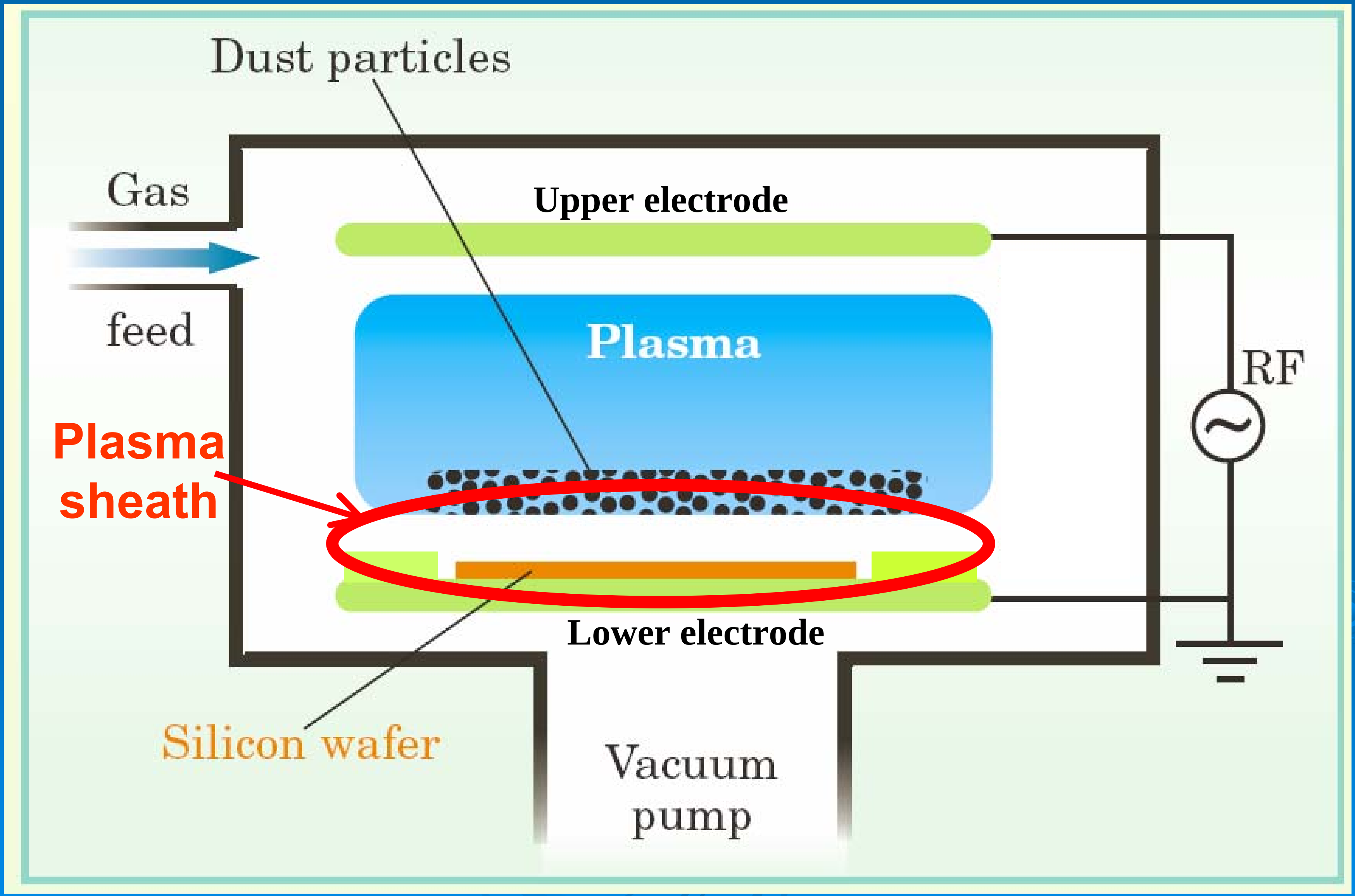
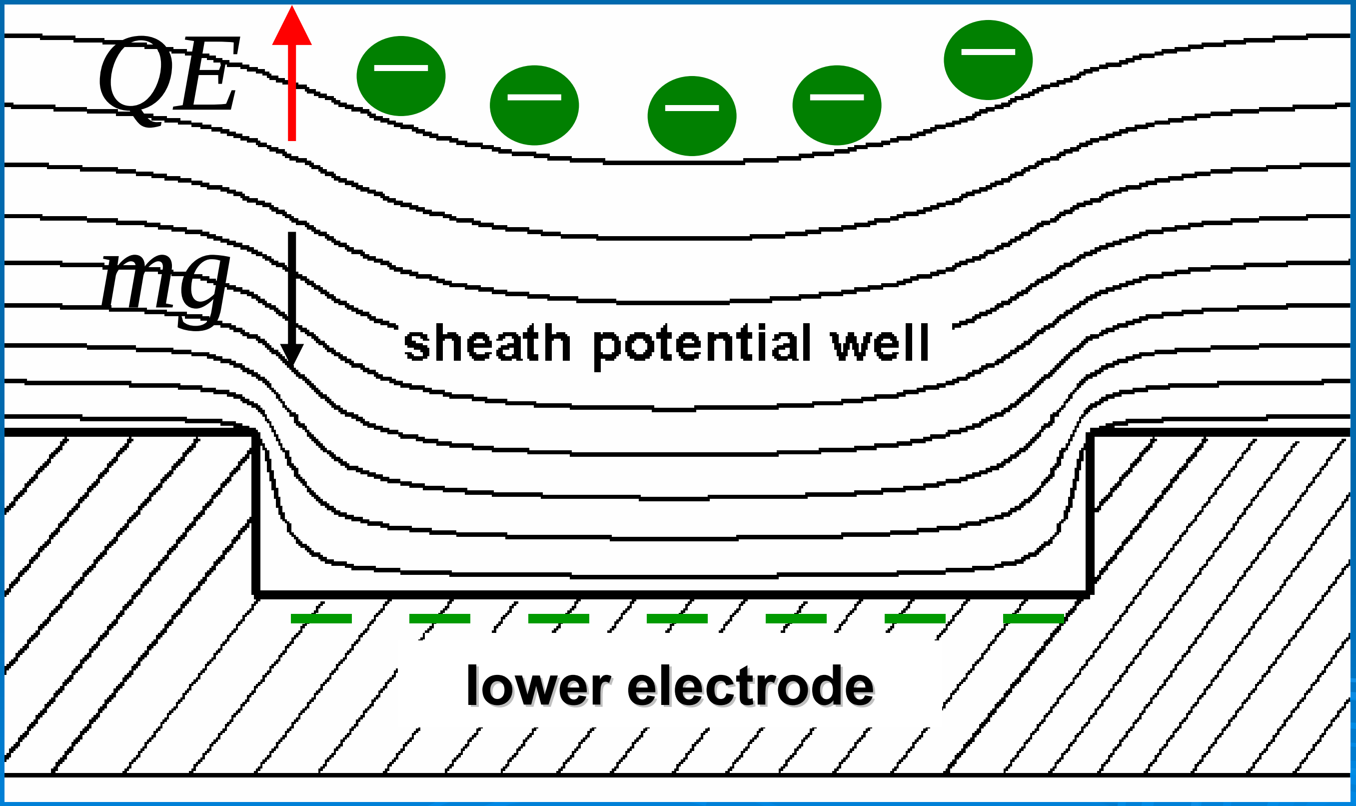
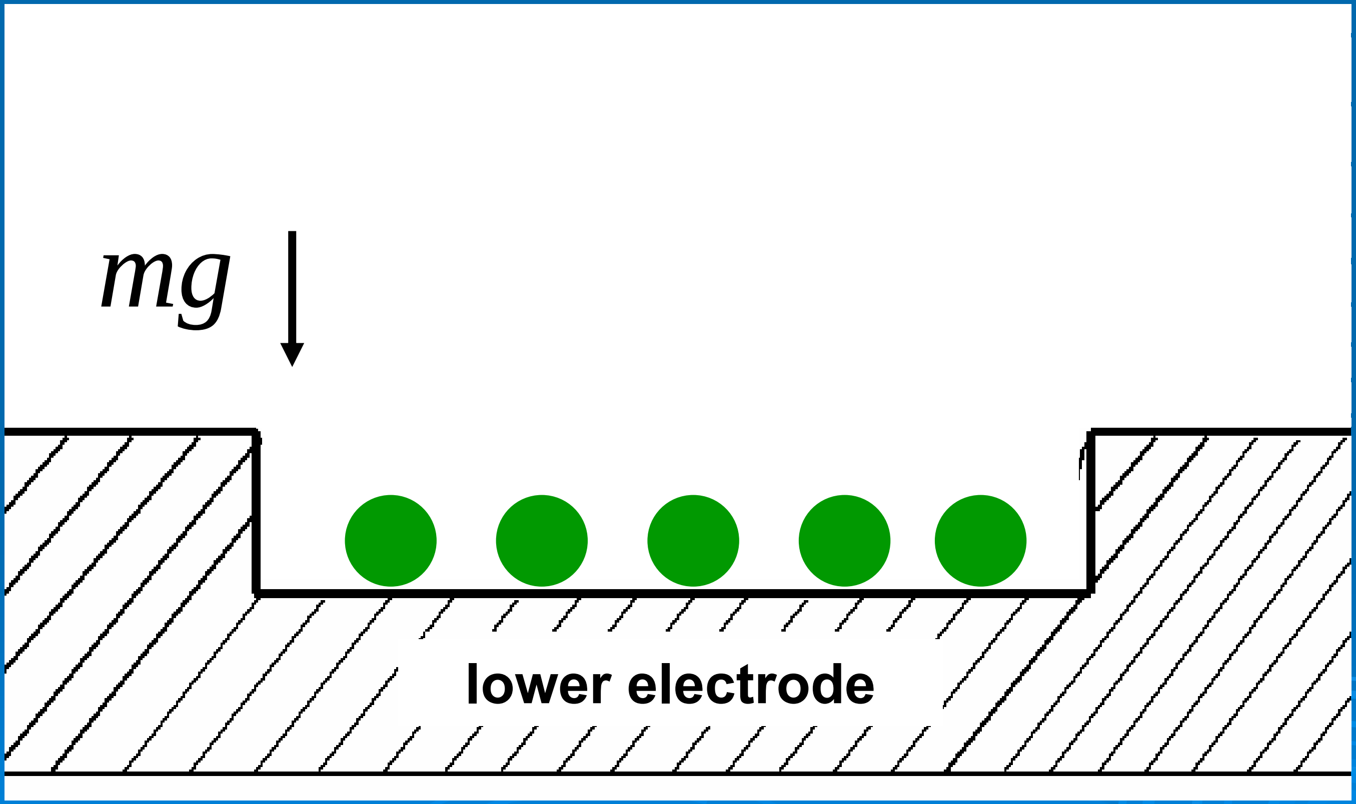


Levitation of dust particles in plasma sheath



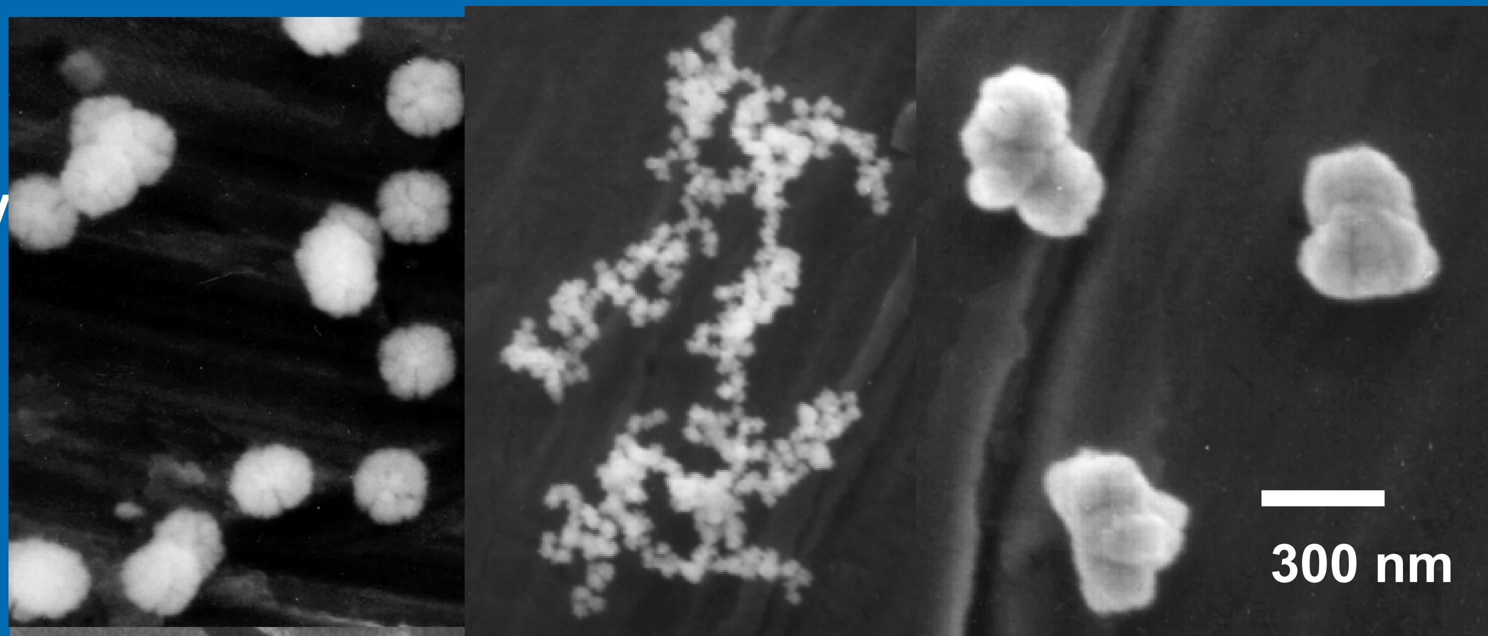
Levitation of dust particles in plasma sheath



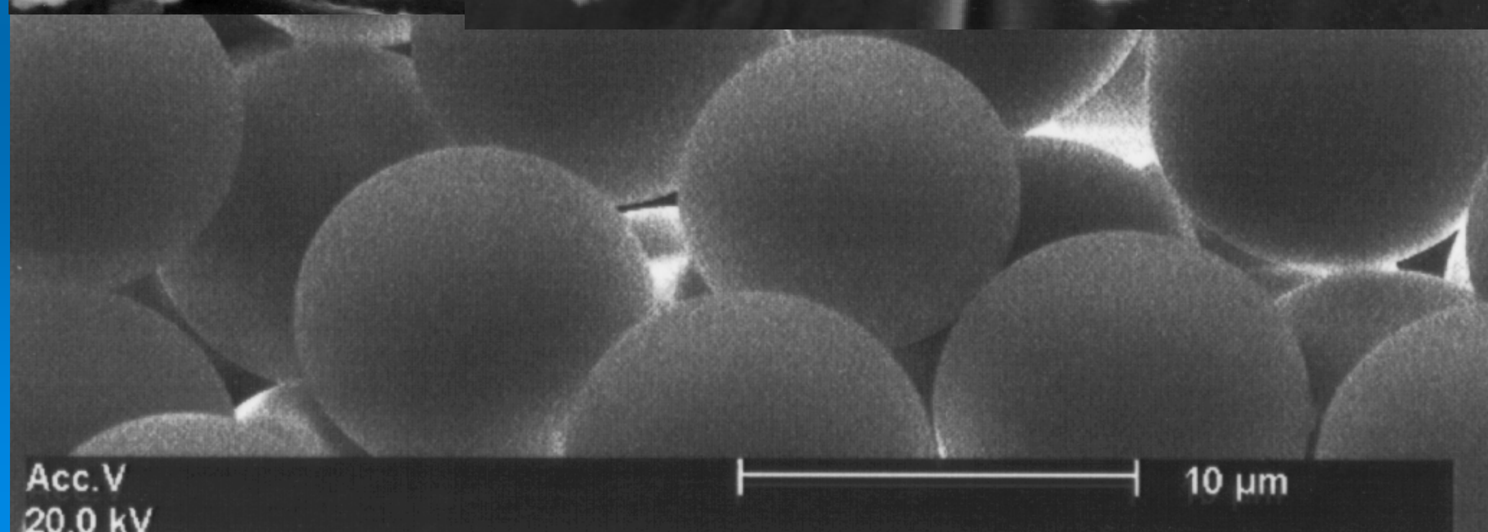
Power off

Examples of dust in laboratory

Grown spontaneously during gas discharge



Purchased from a vendor



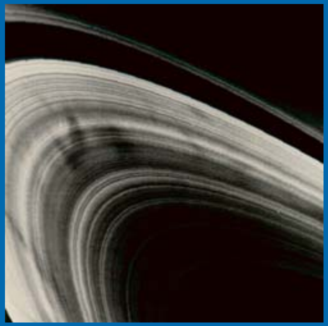
Typical laboratory parameters

- Discharge conditions. gas: Ar, power:~10 W, pressure: 1~X00 Pa, frequency ~10 MHz
- Plasma density: 10^8~10^9 cm^{-3}
- Electron temperature: 1~5 eV
- Size of dust particle: ~1-10 μm
- Inter-particle distance: ~ 0.1-1 mm
- Dust charge (negative) : 10^3~10^4 e
- Mass of dust particle: ~ 10^{-10} g

Why study Dusty Plasma?

Solar system

- Rings of Saturn
- Comet tails

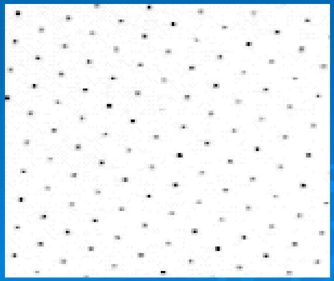


Manufacturing

- Particle contamination
  (Si wafer processing)
- Nanomaterial synthesis

Basic physics

- Coulomb (plasma) crystals
- Waves



Credit: Nosenko *et al.*

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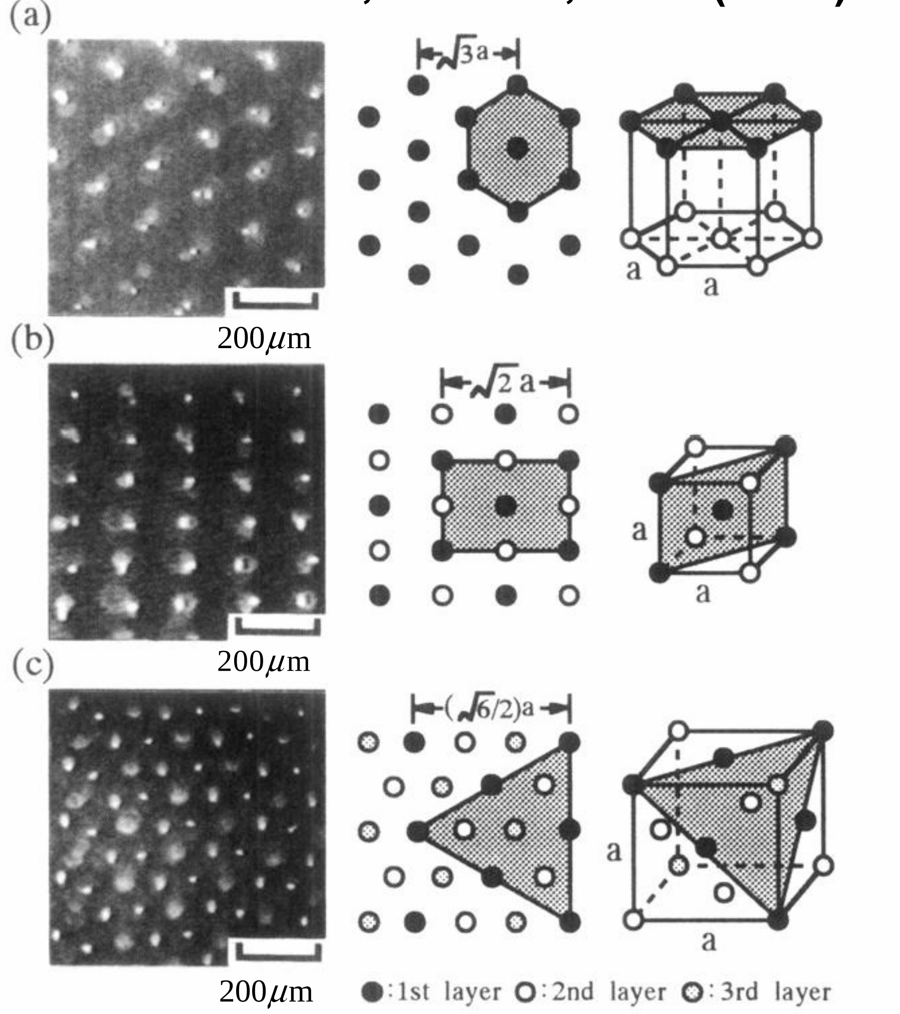
“Discovery” of Plasma Crystals

- Thomas, Morfill, Demmel and Goree et al., Phys. Rev. Lett. 73, 652 (1994).
- Chu and Lin, Phys. Rev. Lett. 72, 4009 (1994).
- Hayashi and Tachibana, Jpn. J. Appl. Phys. 33, L804 (1994).
- Melzer, Trottenberg and Piel, Phys. Lett., A 191, 301 (1994).

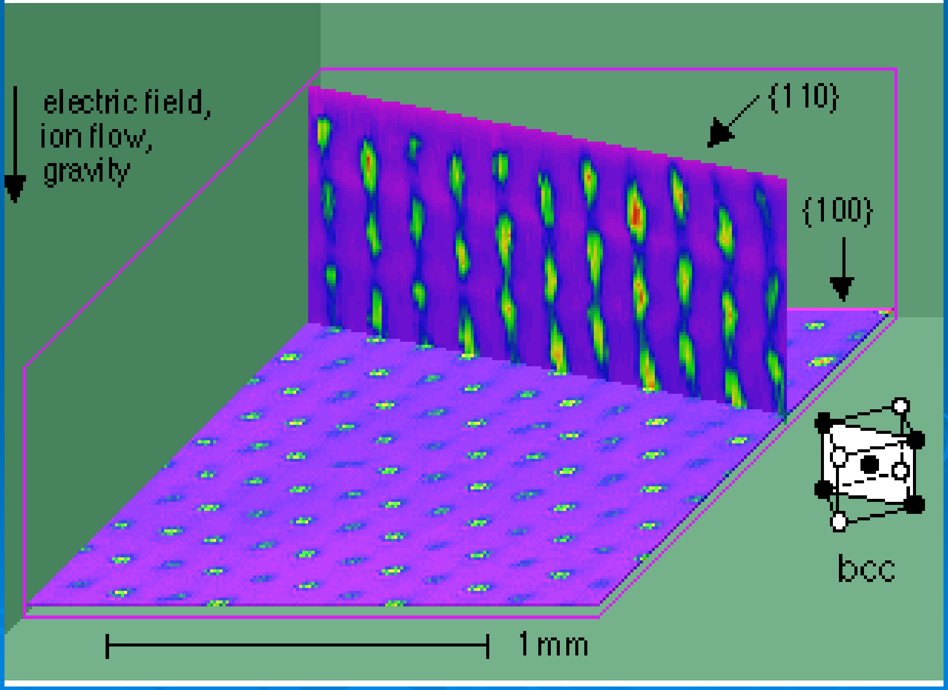
Observations of 3D plasma crystals

Experimental snapshots

Chu and I, PRL. 72, 4009 (1994)

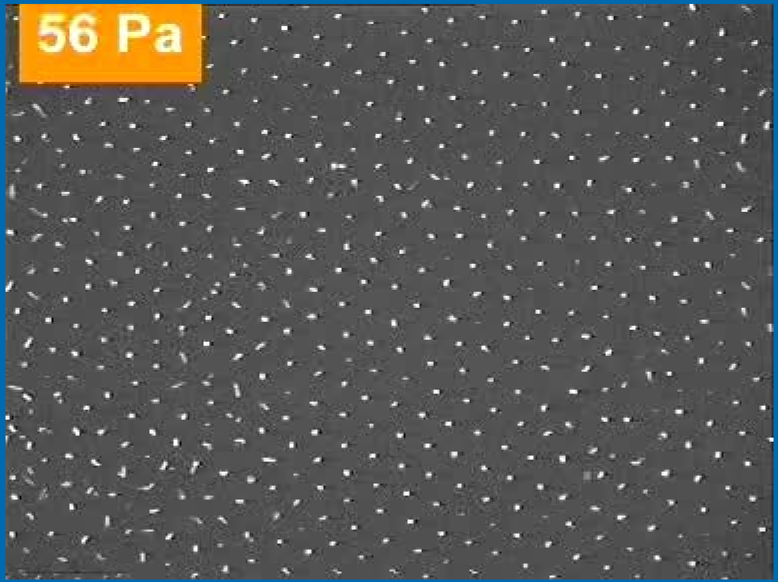


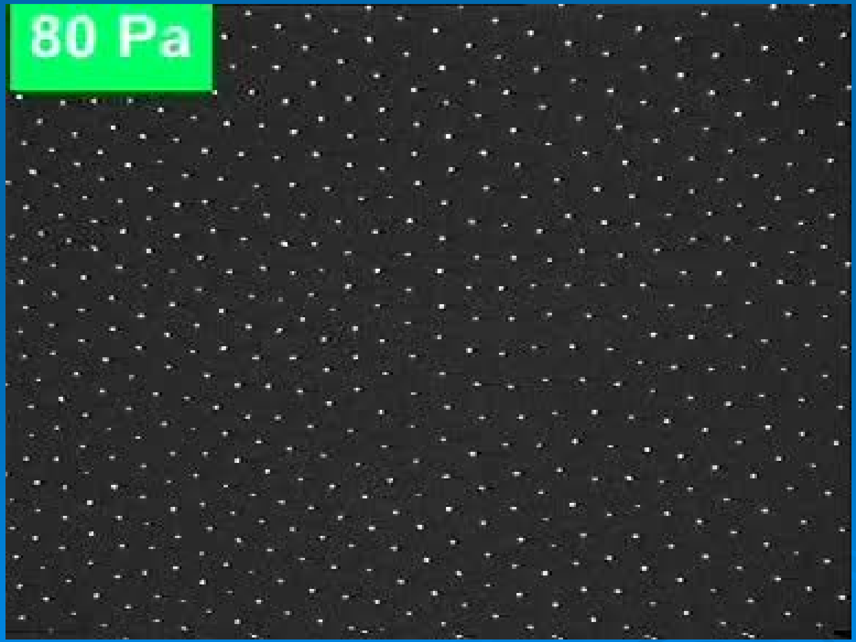
Pieper et al., PRE 54, 5636（1996）

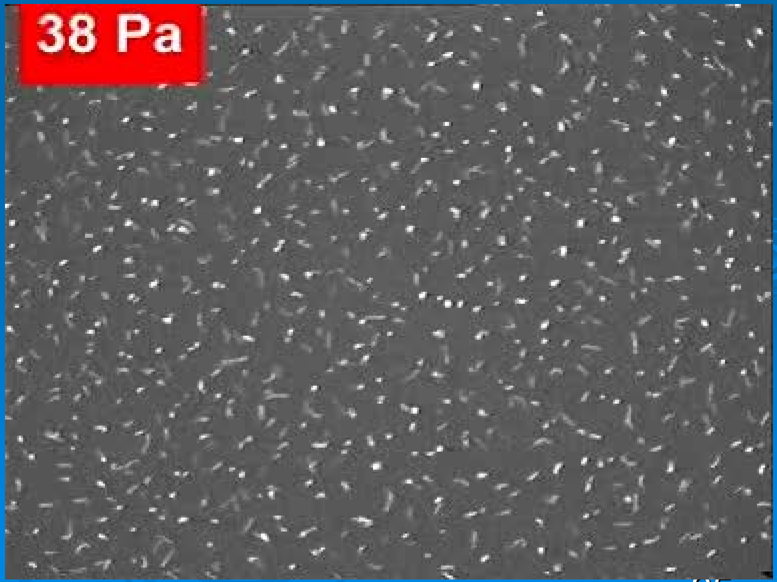


Three states of 2D plasma crystal

Experimental movie of 2D plasma crystal in solid, liquid and gas states

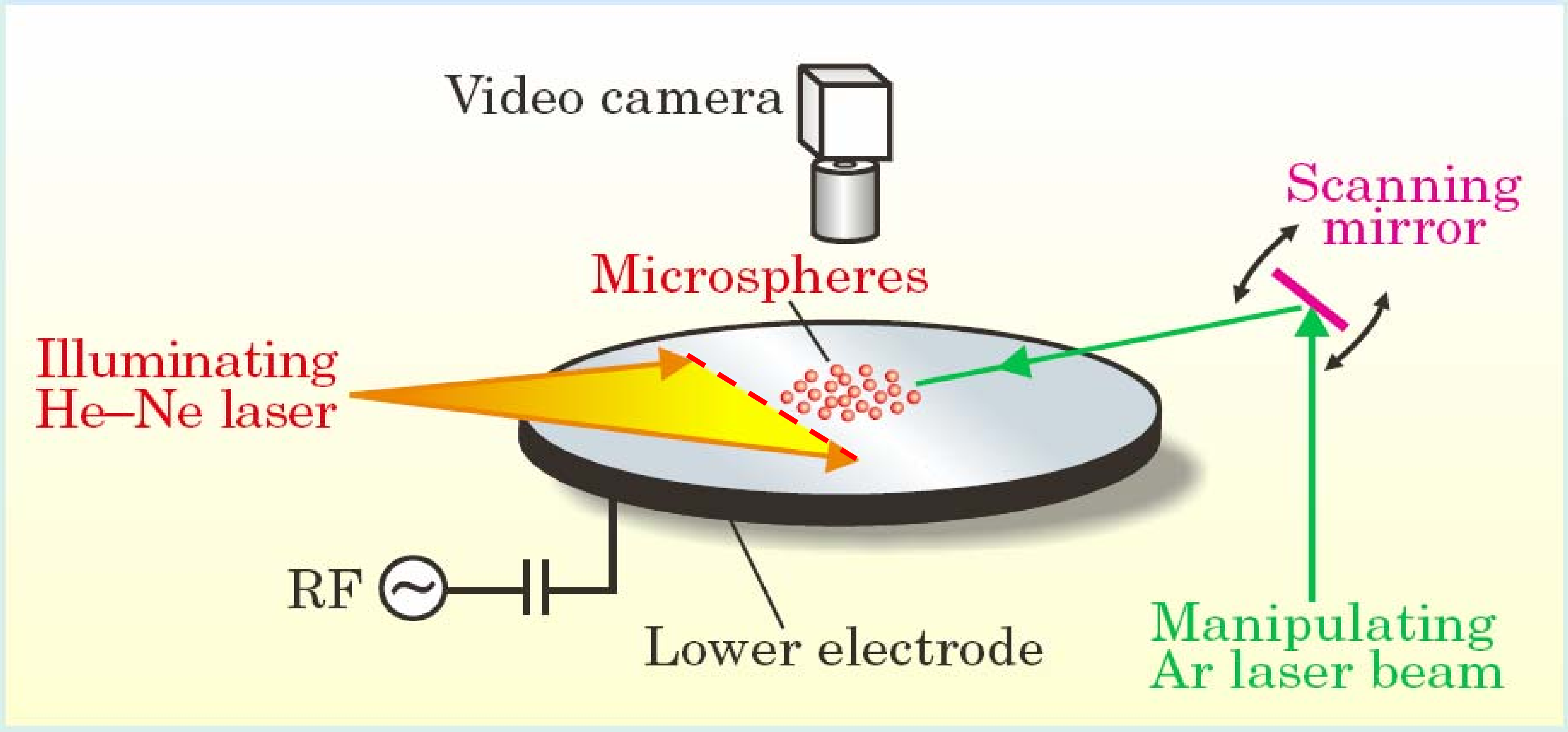




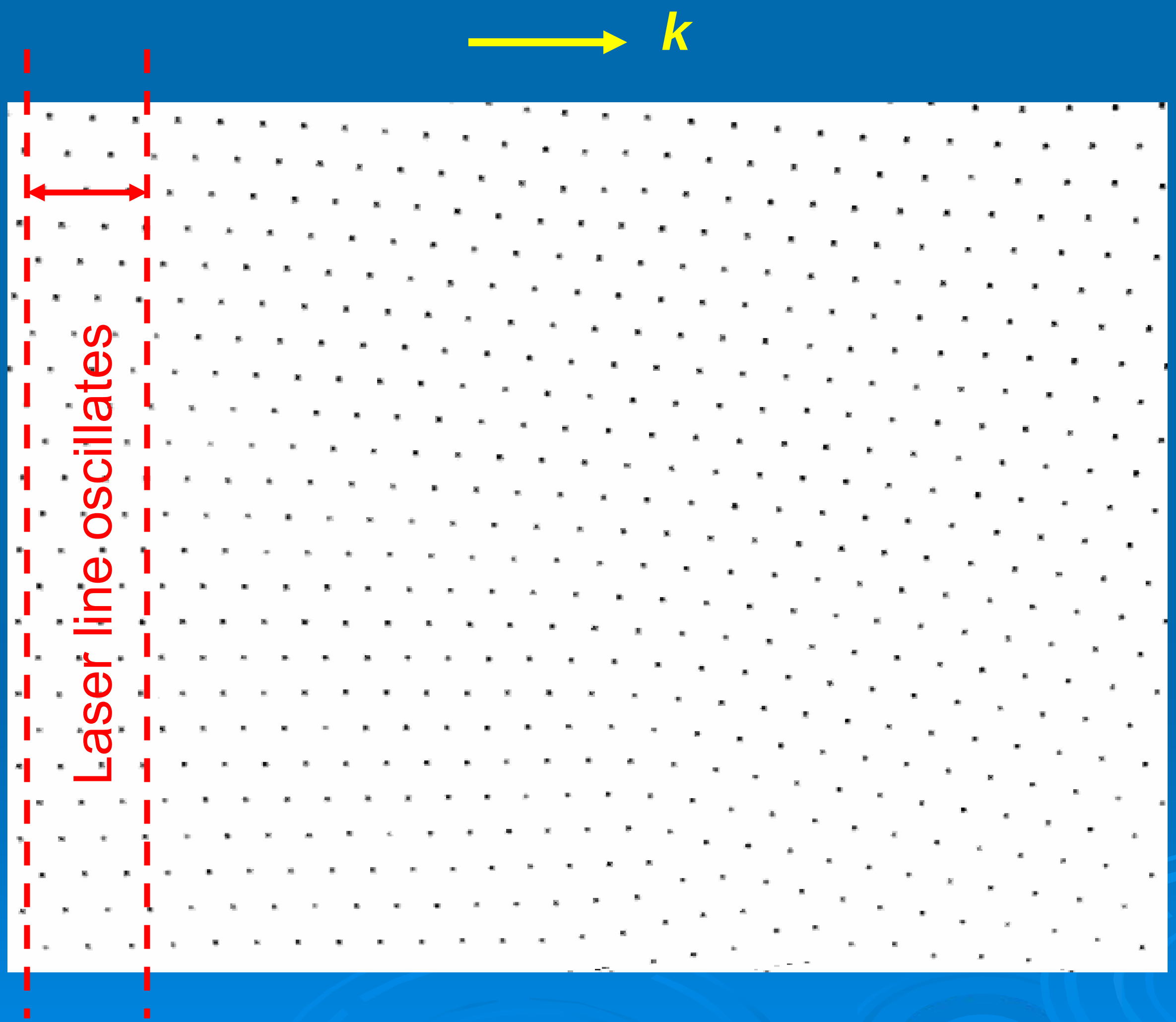


From Complex Plasma Laboratory of Sydney

Excitation of waves in 2D dust crystal: experimental scheme

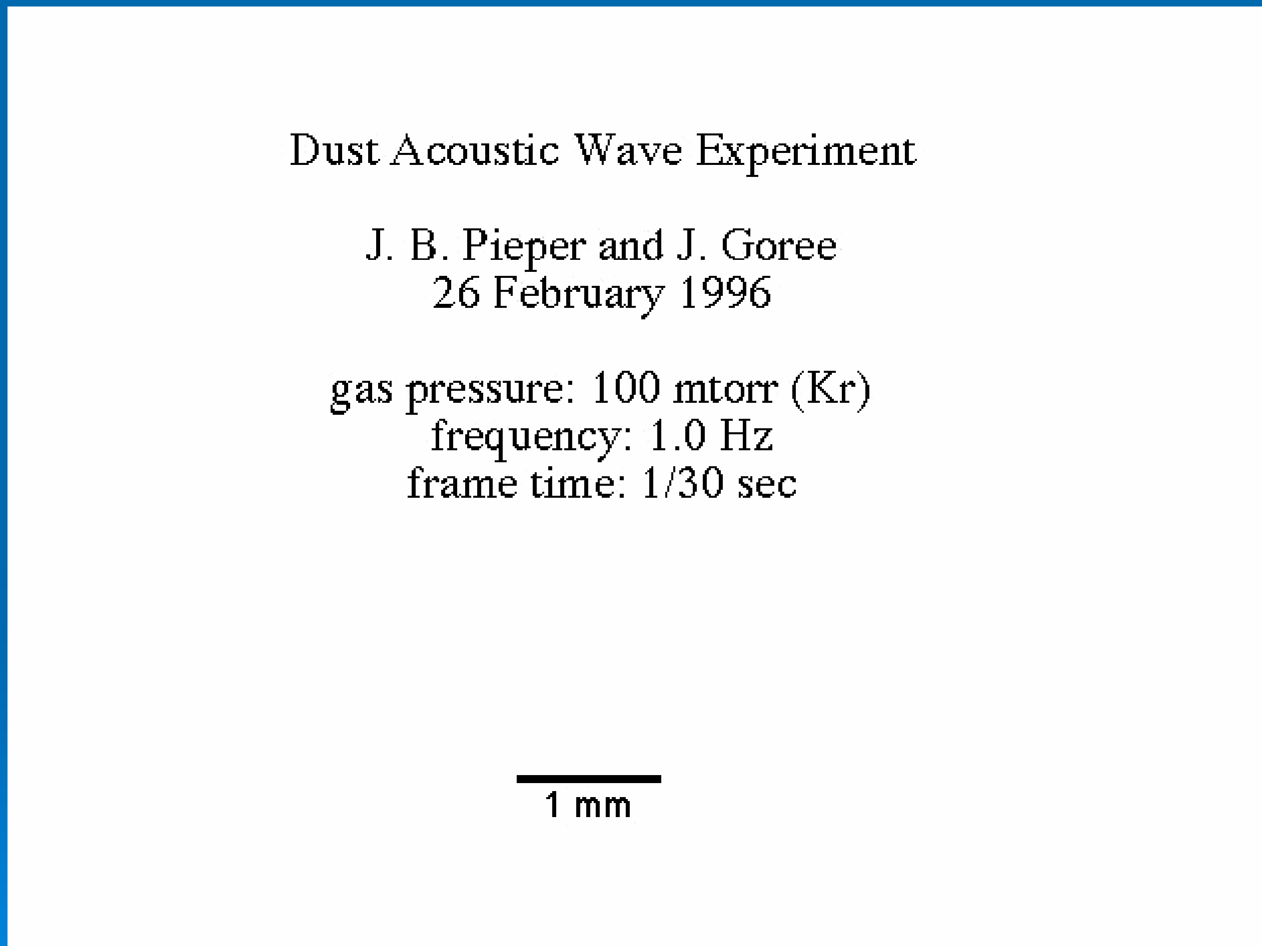


Sinusoidally excited longitudinal wave



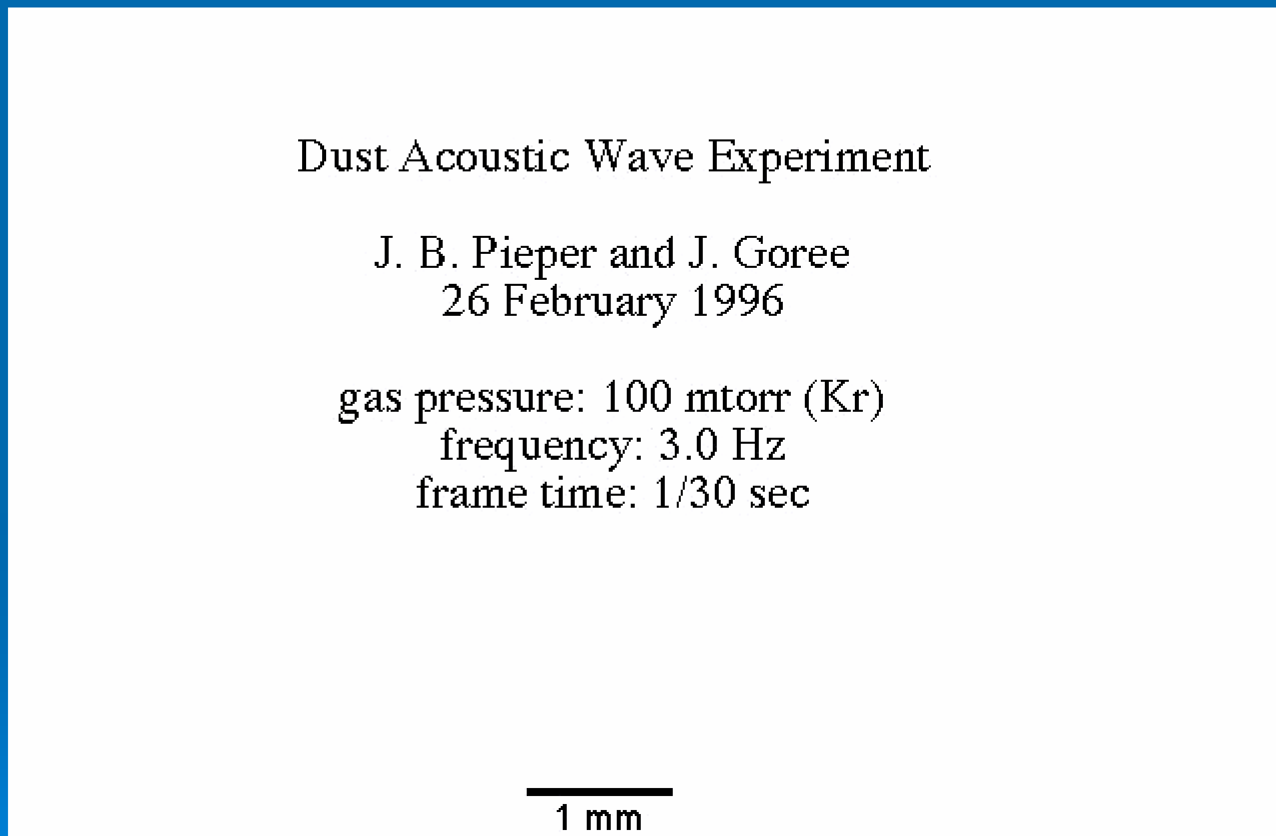
Nunomura *et al.* Phys. Rev. E 2002

Wave processes in dust layers

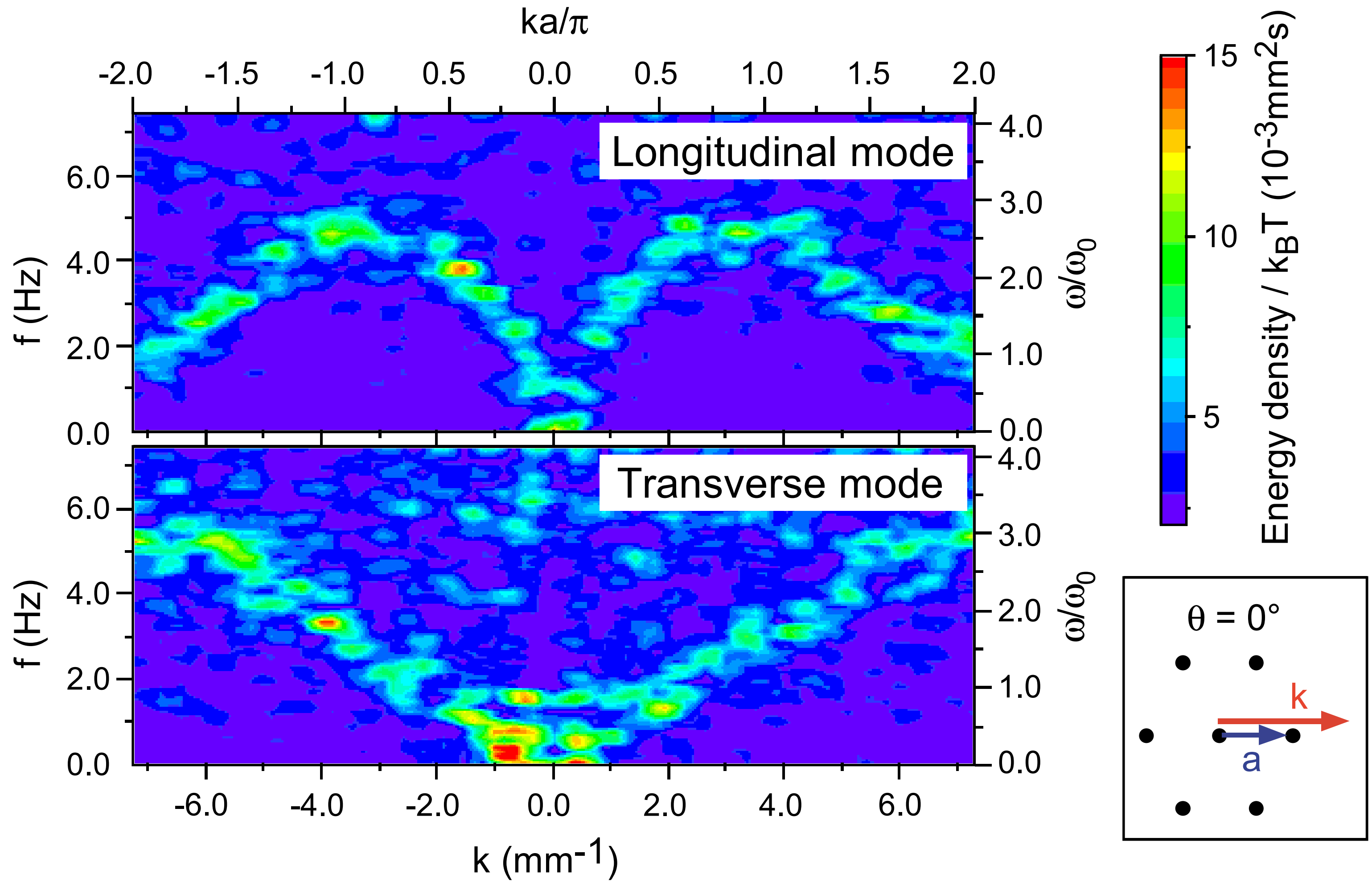


Credit: Goree *et al.*

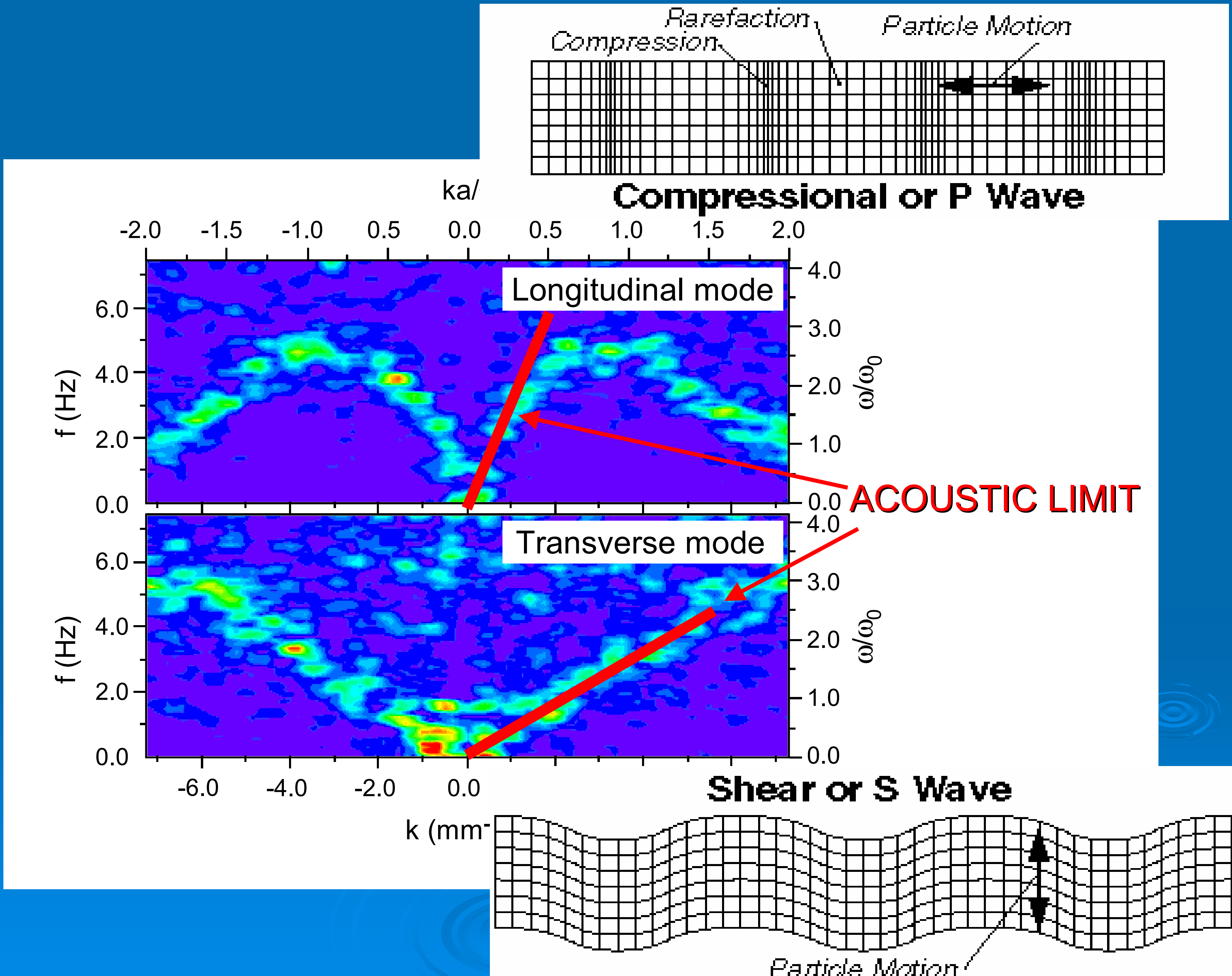
Wave processes in dust layers



Phonon spectrum



Nunomura *et al.* PRL 2002



Rarefaction
Compression
Particle Motion
Compressional or P Wave
ka/
-2.0
-1.5
-1.0
0.5
0.0
0.5
1.0
1.5
2.0
Longitudinal mode
6.0
4.0
2.0
0.0
f (Hz)
4.0
3.0
2.0
1.0
0.0
ω/ω0
ACOUSTIC LIMIT
Transverse mode
-6.0
-4.0
-2.0
0.0
k (mm
Shear or S Wave
Particle Motion

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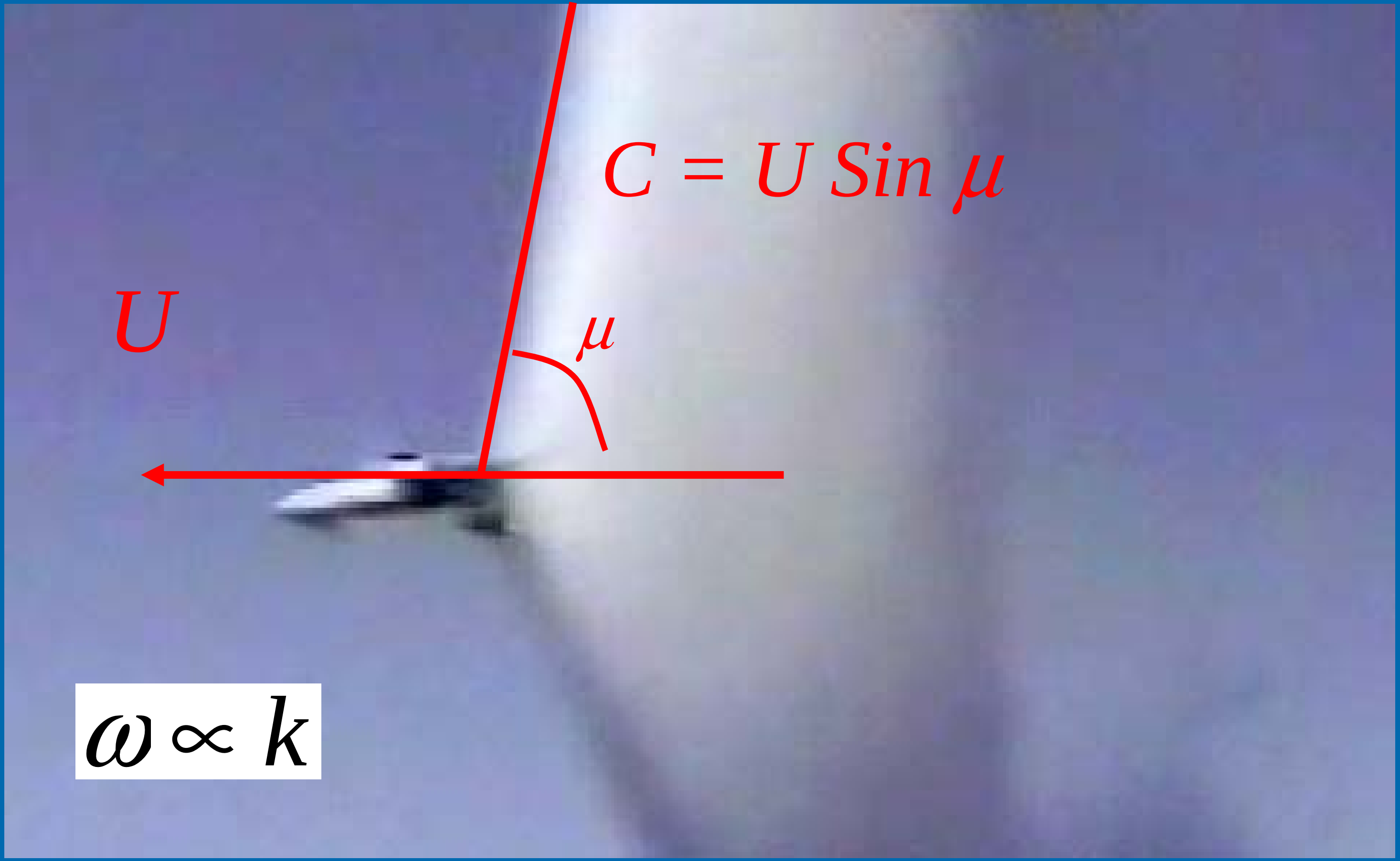
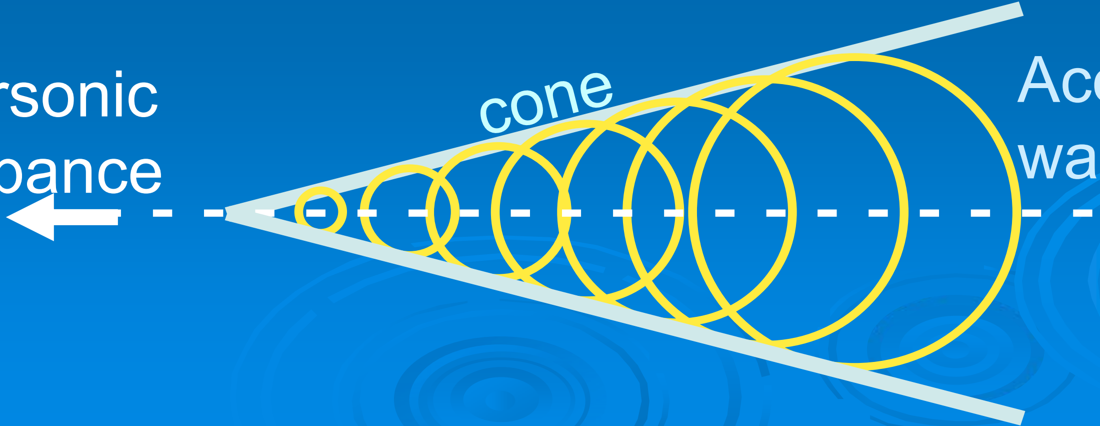
Mach cones

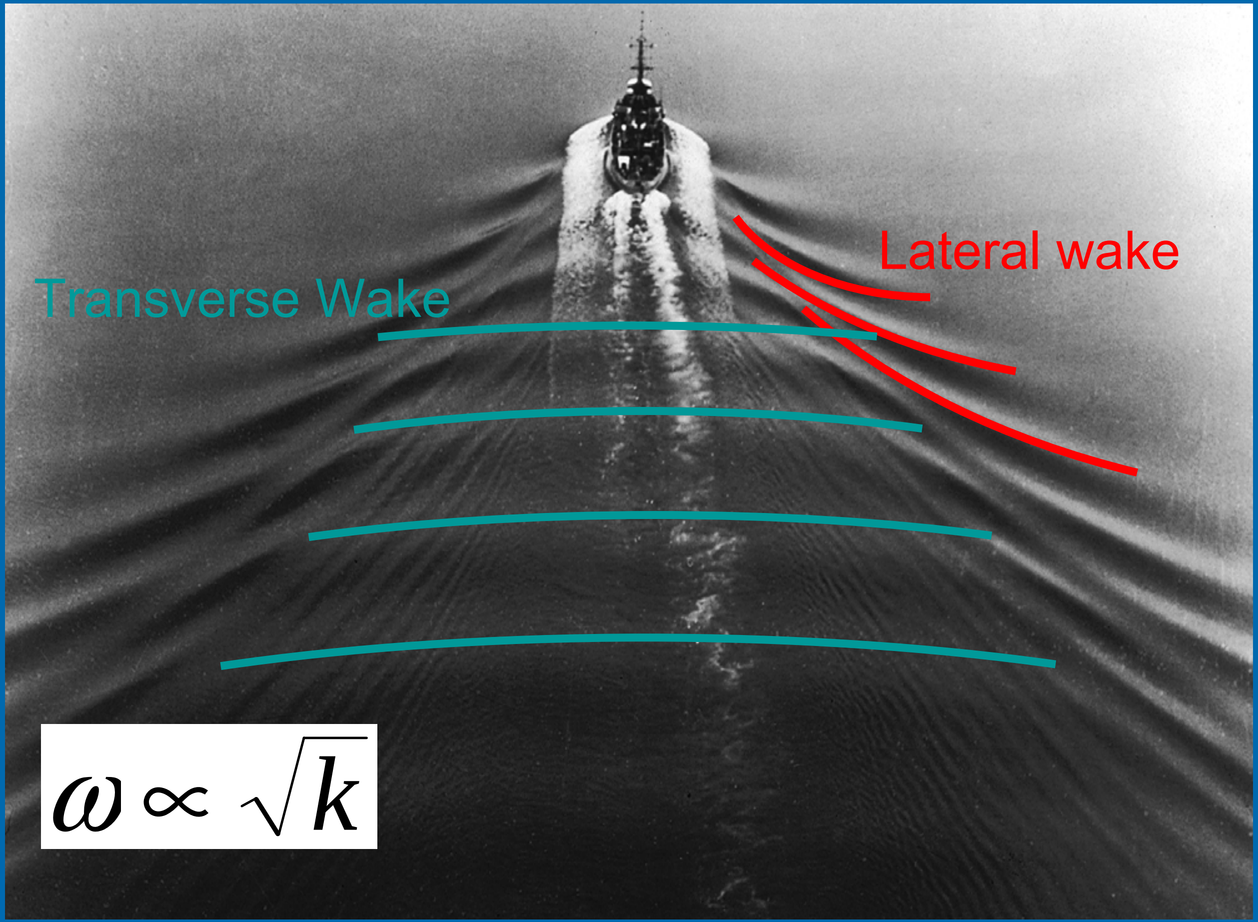
Credit: Goree

Mach cone angle



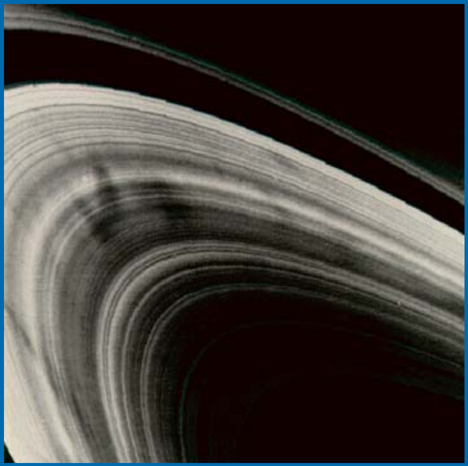


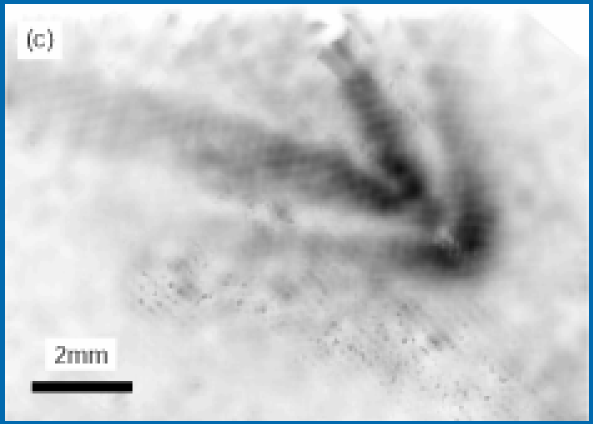
Ship's wake

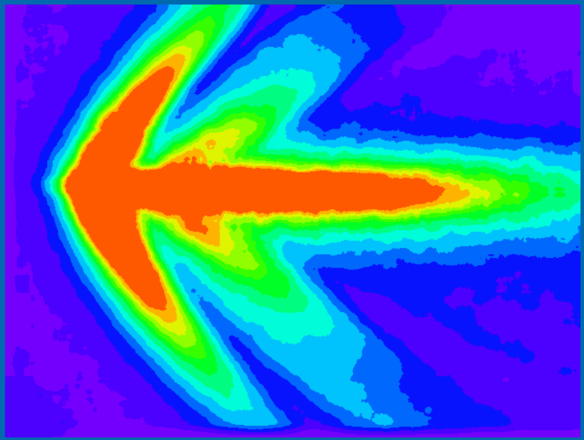


Credit: Goree

Mach cones in dusty plasmas







Havnes 1995

Existence predicted theoretically, for Saturn's rings

Samsonov 1999

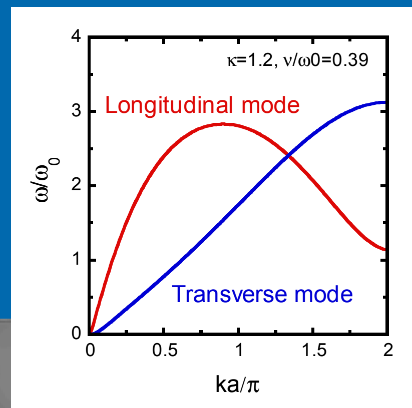
Discovered experimentally in lab, by external charged particle

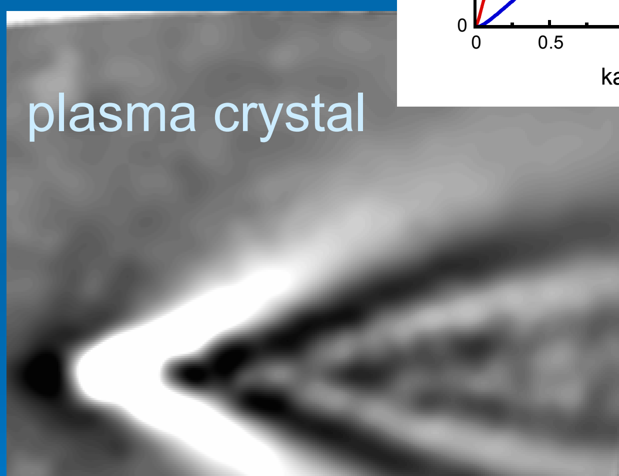
Melzer & Nunomura 2000

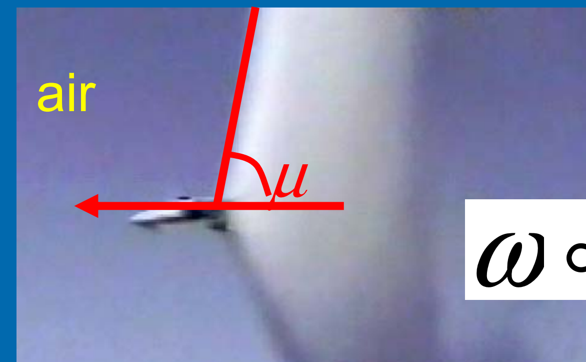
Excitation of Mach cones in lab, by moving laser spot

Credit: Nosenko *et al.*

Wake pattern determined by dispersion relation







$$\omega \propto k$$

Mach cone

$$\omega \not\propto k$$

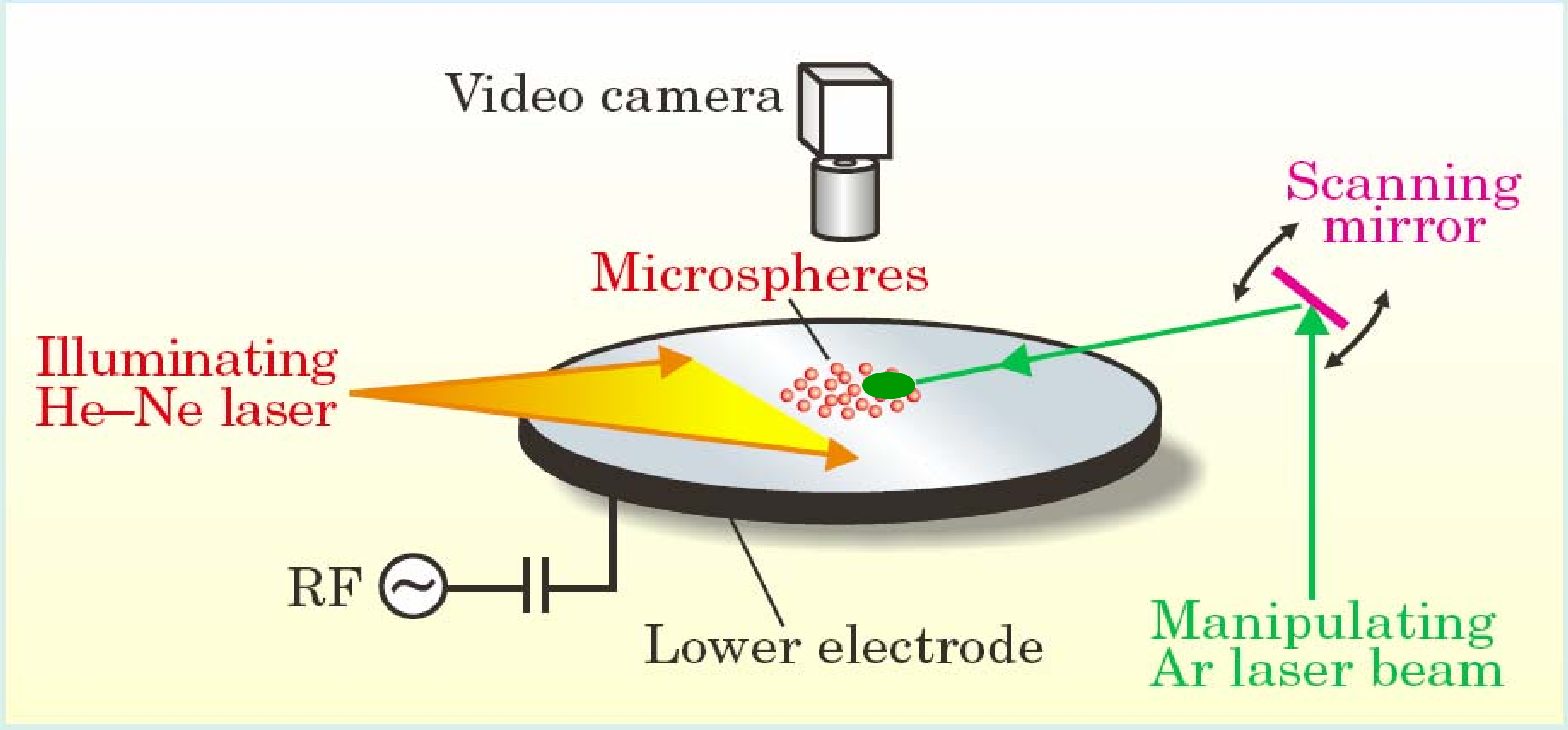
Has both features:

- Mach Cone
- Lateral & transverse wakes

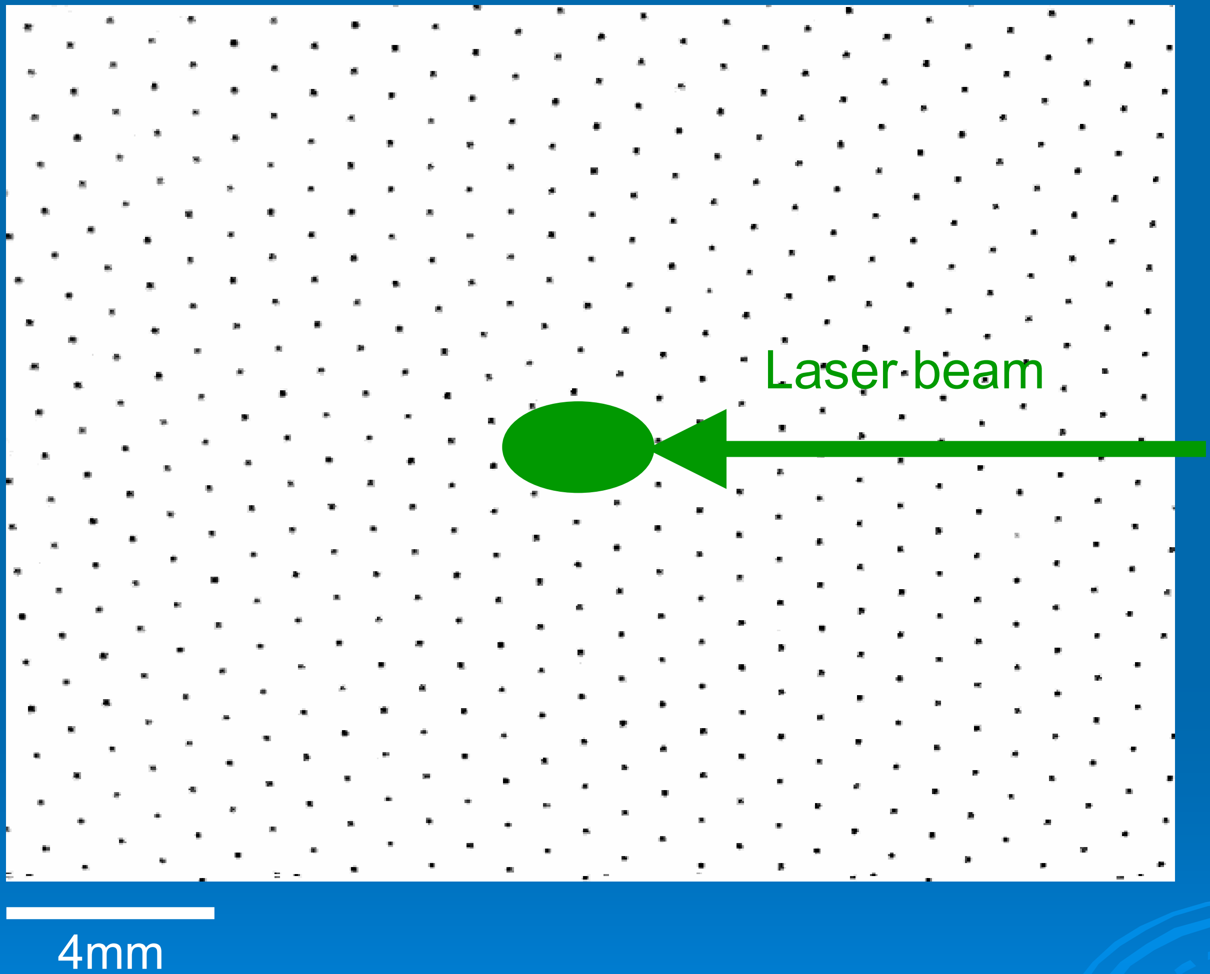
Lateral & transverse wakes

Credit: *Goree*

Mach cone by moving laser spot: experimental scheme



Mach cone excitation by laser spot

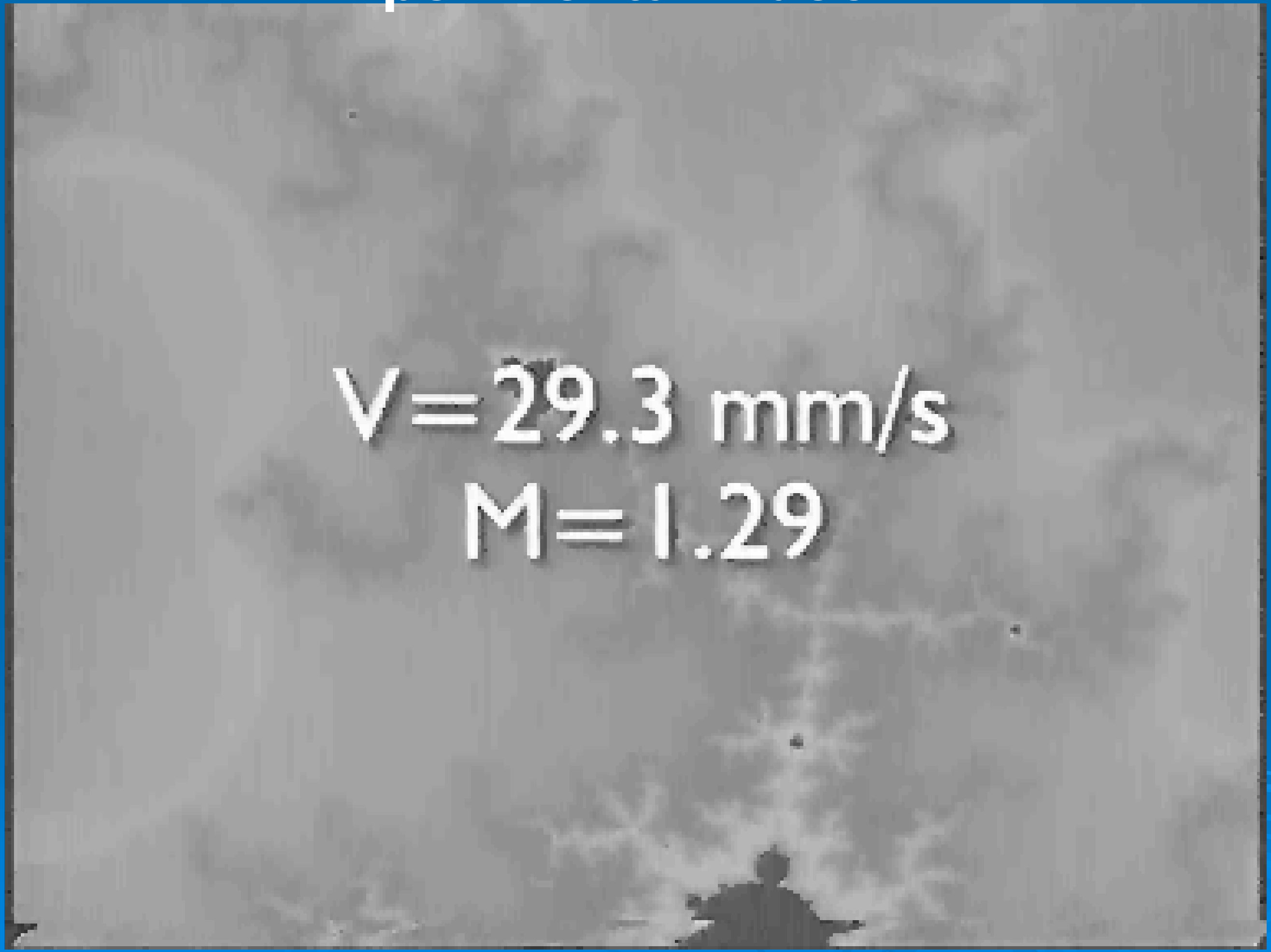


$M=V/C_L = 1.29$

Nosenko *et al.* PRL 2002

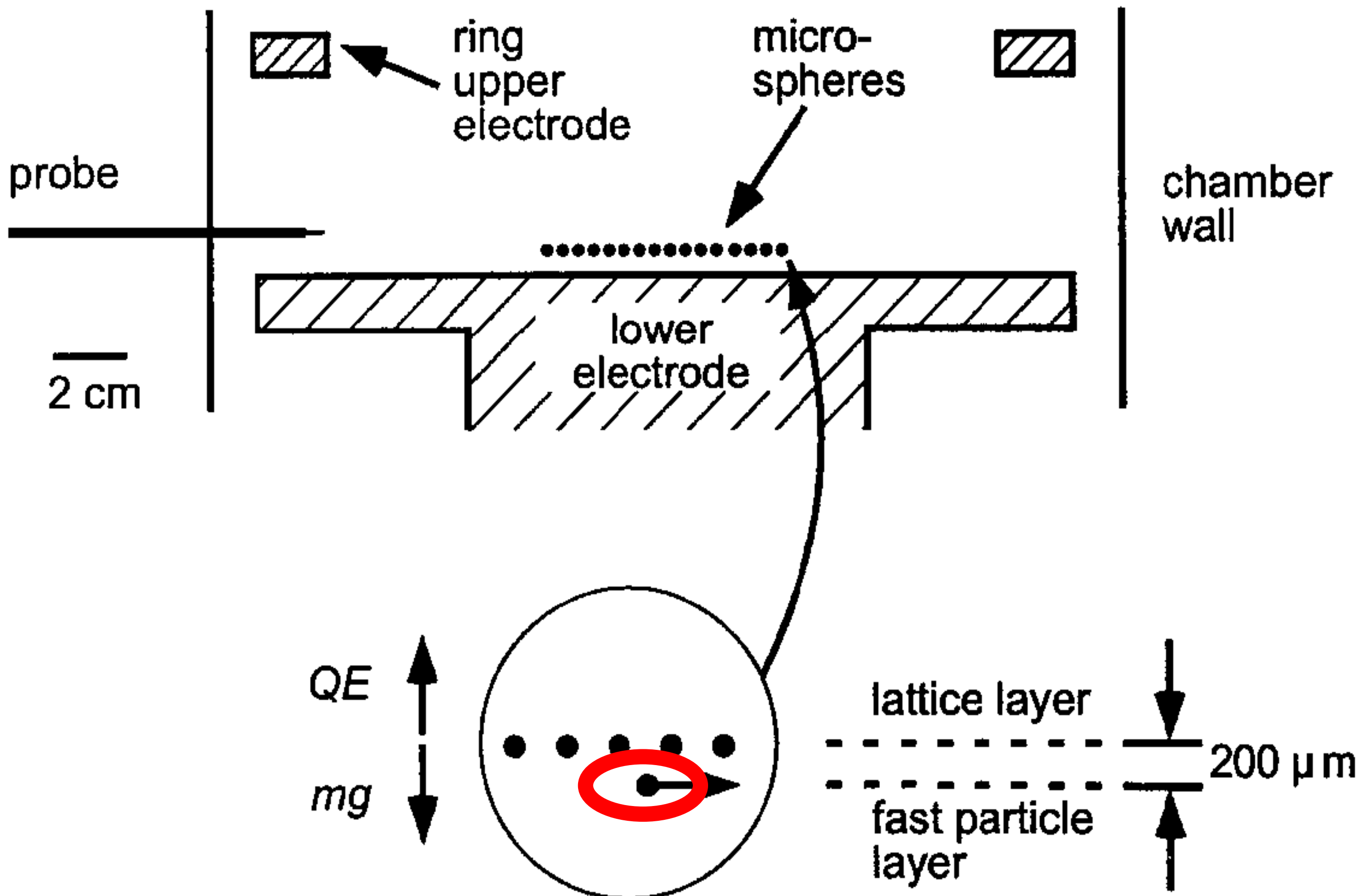
Mach cone in 2D Dusty Plasma

Experimental video

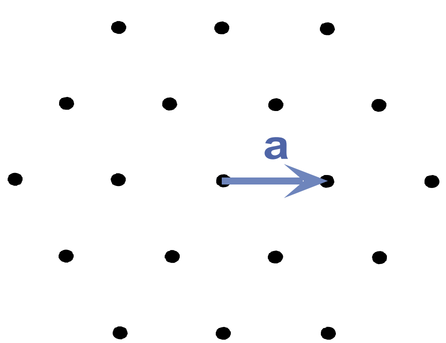


A. Melzer, S. Nunomura, D. Samsonov, Z. W. Ma, and J. Goree, Phys. Rev. E 62, 4162 (2000)

Mach cone by moving external charged particle: experiment

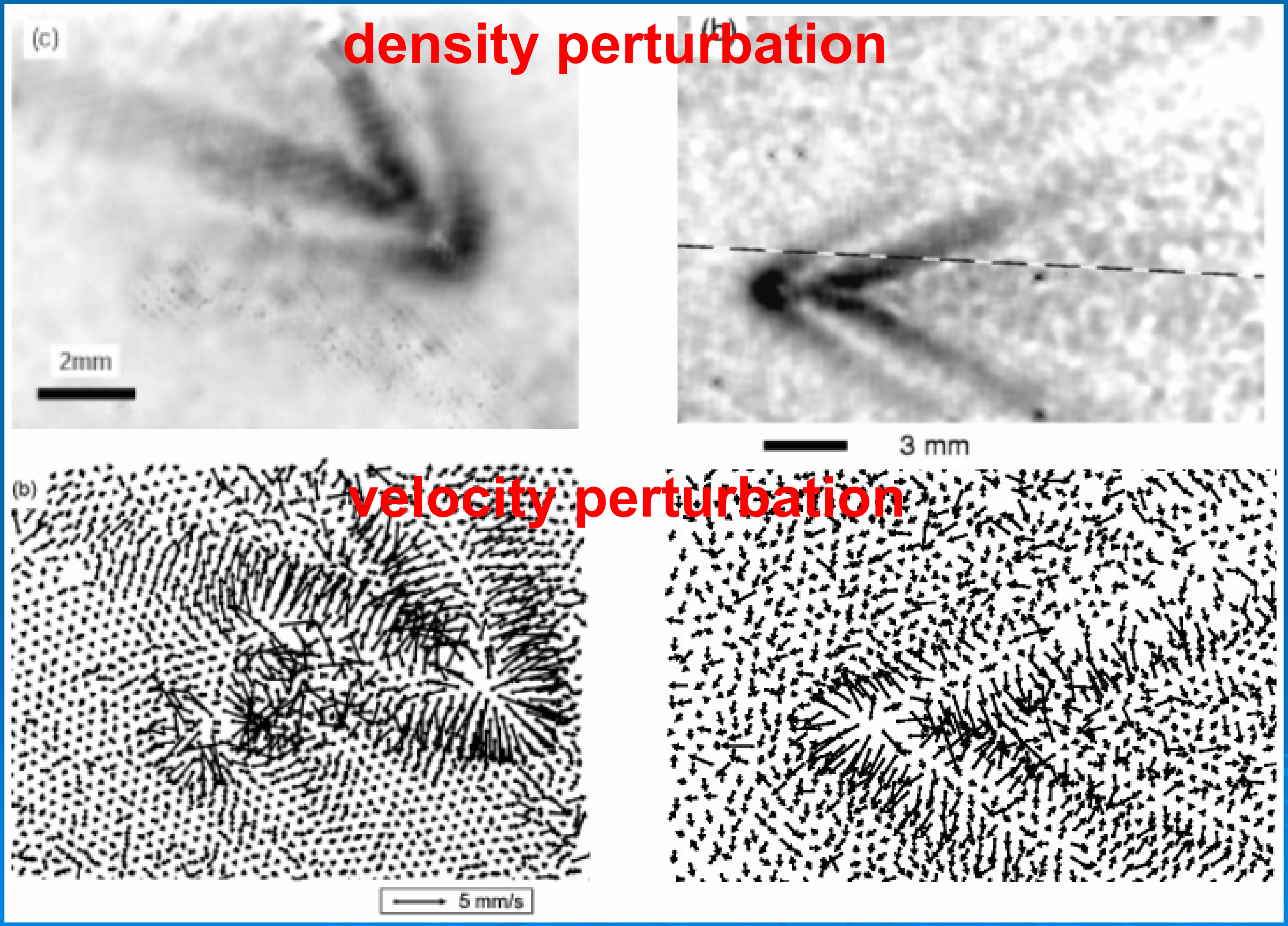


Top view



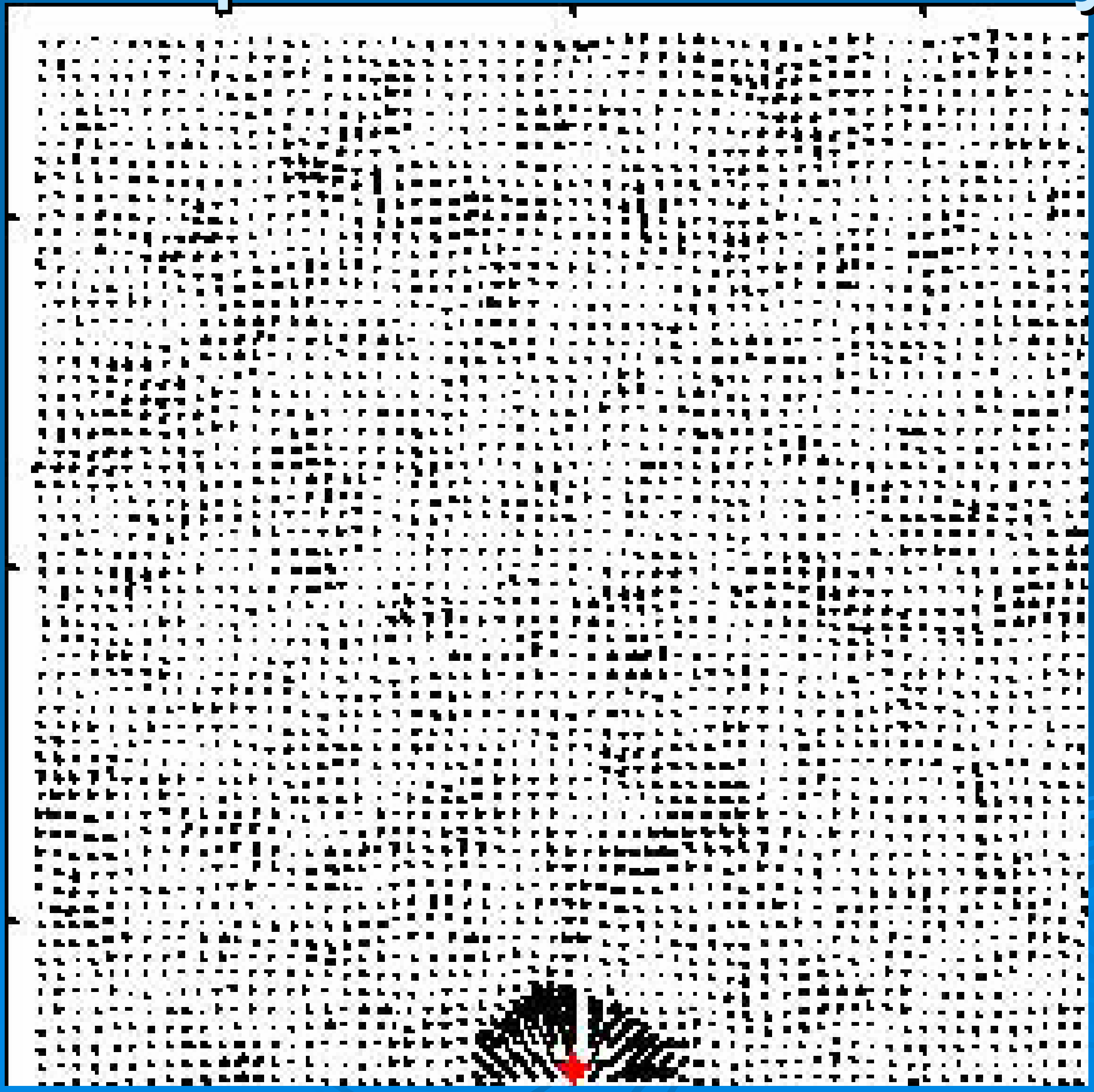
Samsonov, et al., Phys. Rev. Lett. 83, 3649 (1999)

Experimental snapshots



Samsonov, et al., Phys. Rev. Lett. 83, 3649 (1999)

Mach cone excited by external charge: experimental velocity field



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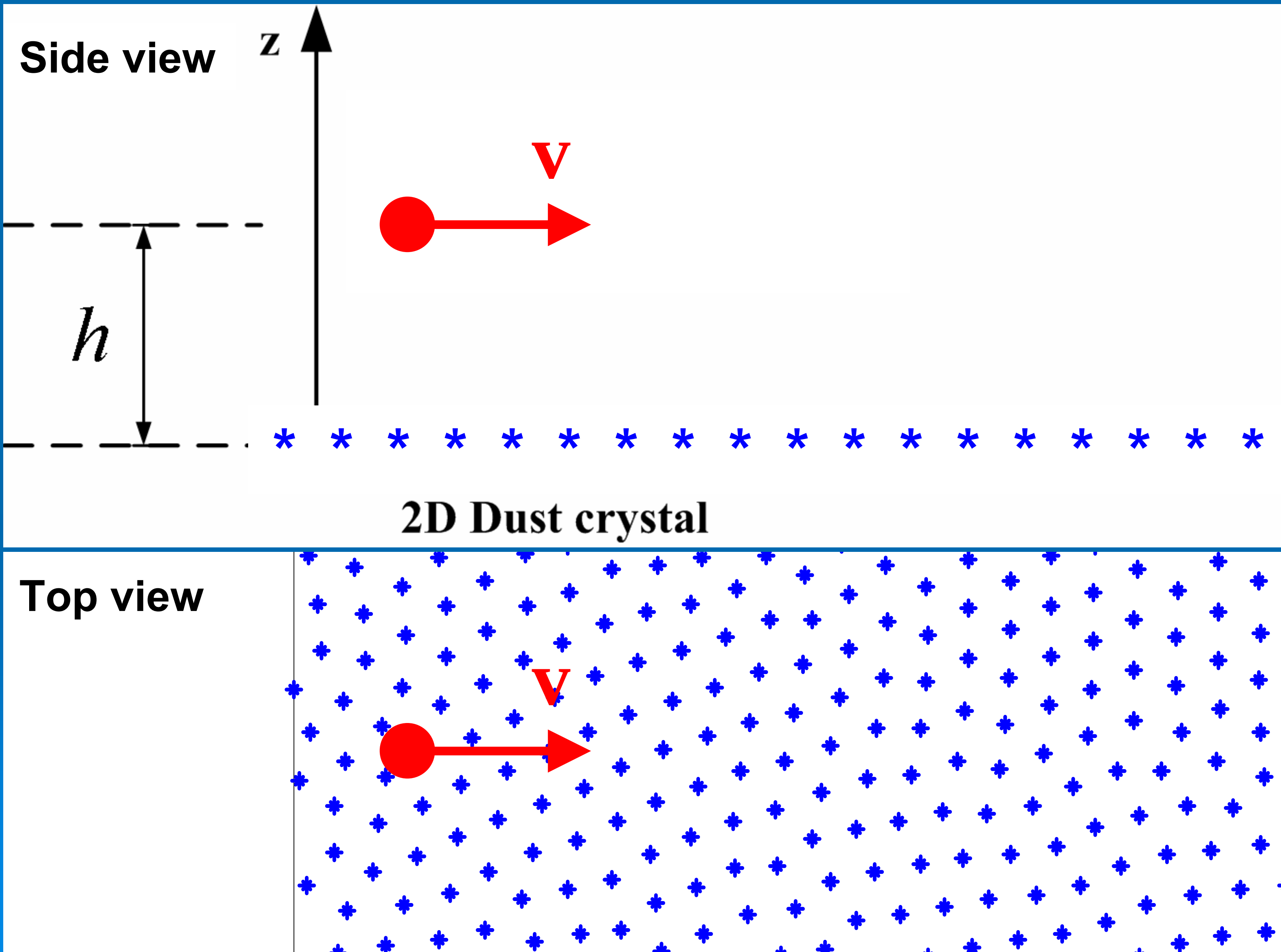
Problem definition



Physical model: 2D Yukawa system

- Electrons and ions provide Debye screening
- Neutral particles provide damping and determine the system temperature
- Dust particles undergo Brownian motion and interact with each other via Yukawa potential

Physical model: 2D Yukawa system

$\kappa = a / \lambda_D$ Screening parameter

$\Gamma = \dfrac{Q^2 / a}{k_B T}$ Interparticle coupling coefficient

γ Damping coefficient

Typically Γ » 1 → strongly coupled system

Experimental support:
Konopka, Morfill, and Ratke, Phys. Rev. Lett. 84, 891 (2000)
Hebner, Riley, Johnson, Ho, and Buss, Phys. Rev. Lett. 87, 235001 (2001).

Problem definition

- Perturbation of the dust layer
  - Polarization
  - Mach cone excitation
- Induced forces acting on test particle
  - Stopping force
  - Image force
  - Transverse forces

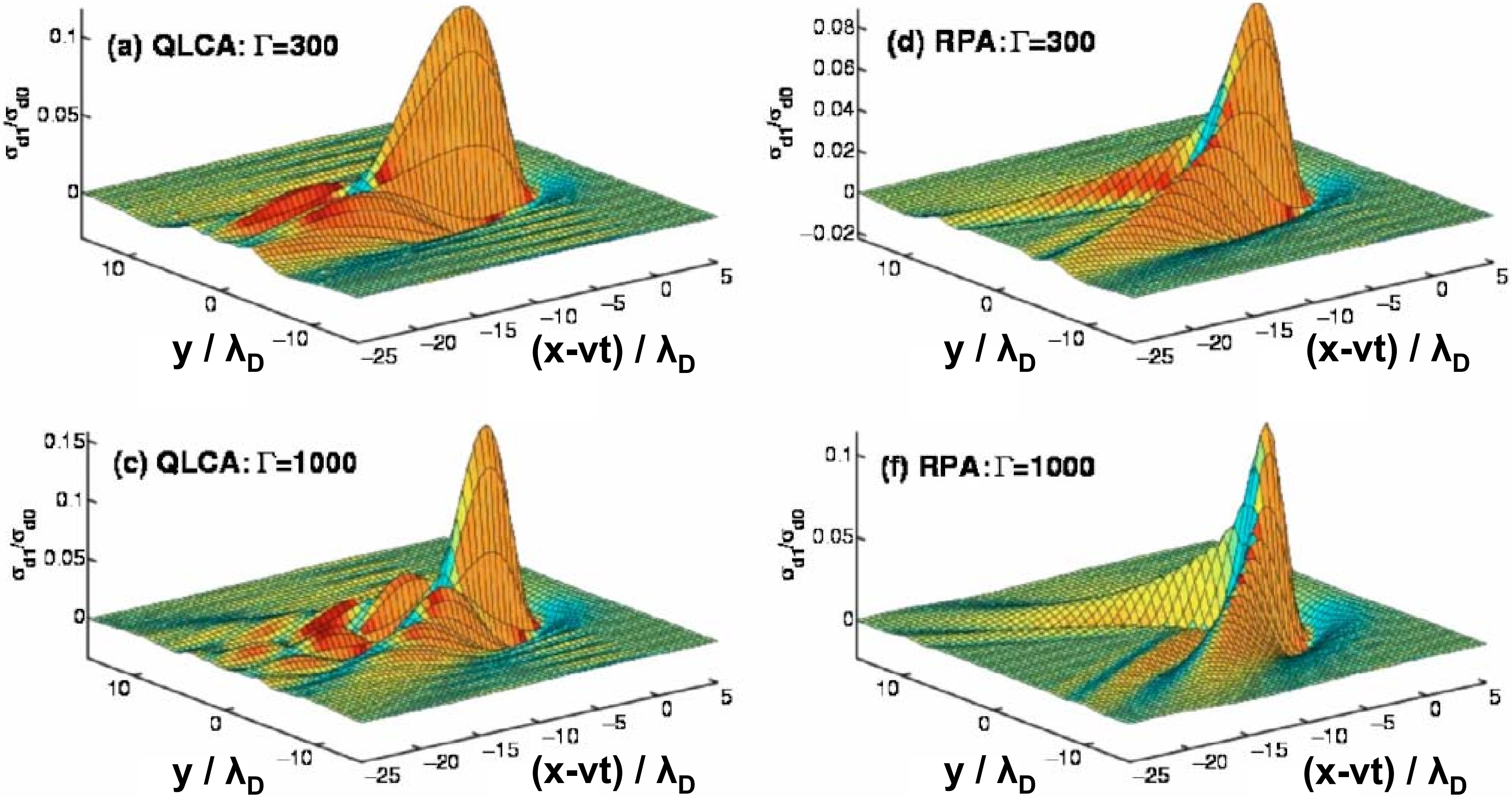
Analytical models

- Random-Phase-Approximation (RPA)
- Quasi-Localized Charge Approximation (QLCA)
  - Kalman and Golden, PRA 41, 5516 (1990)

Computer simulation

- Brownian Dynamics (BD)

Induced dust density: QLCA vs. RPA



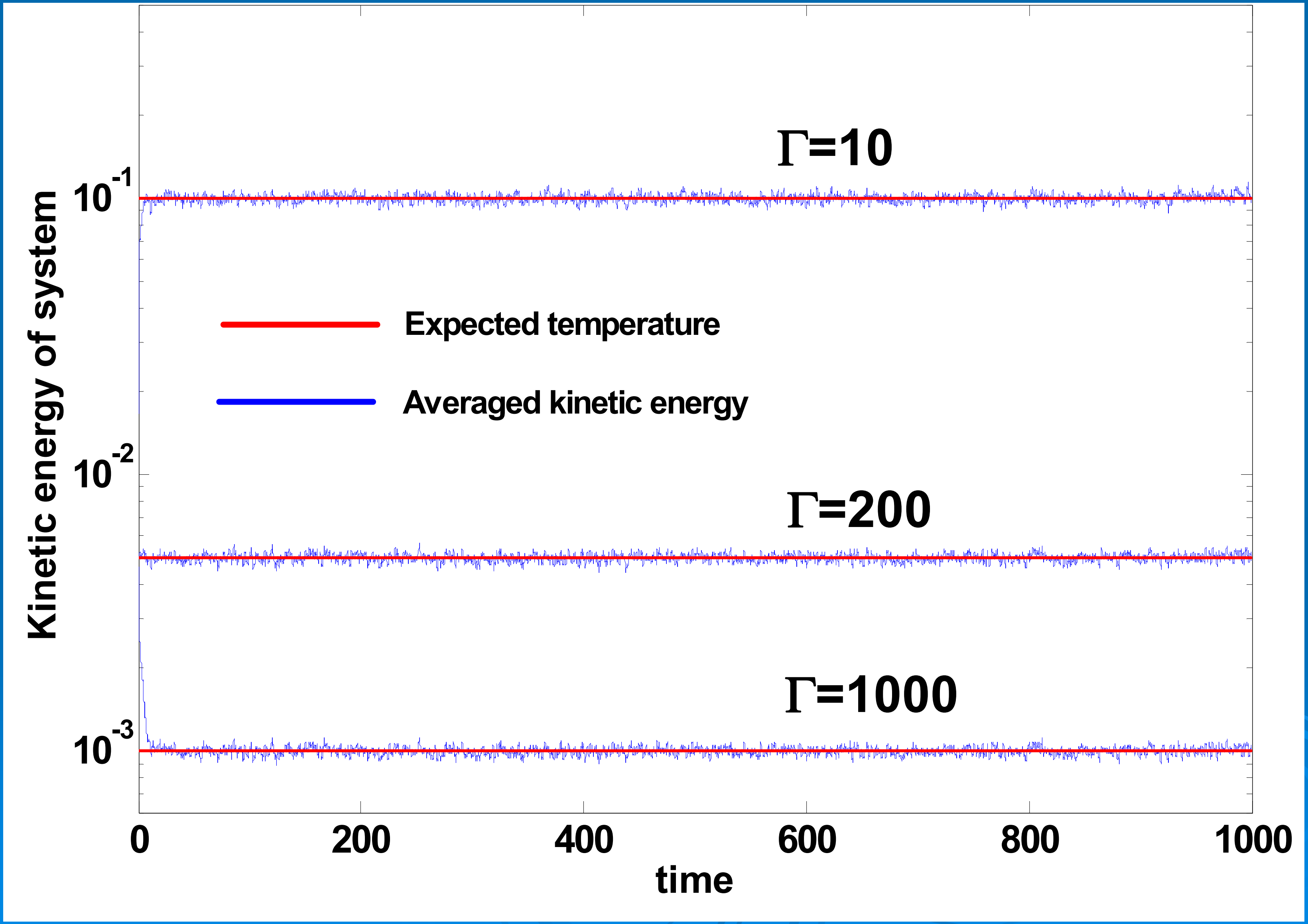
Algorithms for BD simulation

- Euler-like
  - Ermak, J. Chem. Phys. 62, 4189 (1975)
- Beeman-like:
  - Allen, Mol. Phys., 66, 3039 (1980)
- Verlet-like
  - Van Gunsteren and Berendsen, Mol. Phys. 45, 637 (1982)
- Gear-Like Predictor-Corrector
  - Hou, Miskovic and Wang, in preparation

Simulations provide information on:

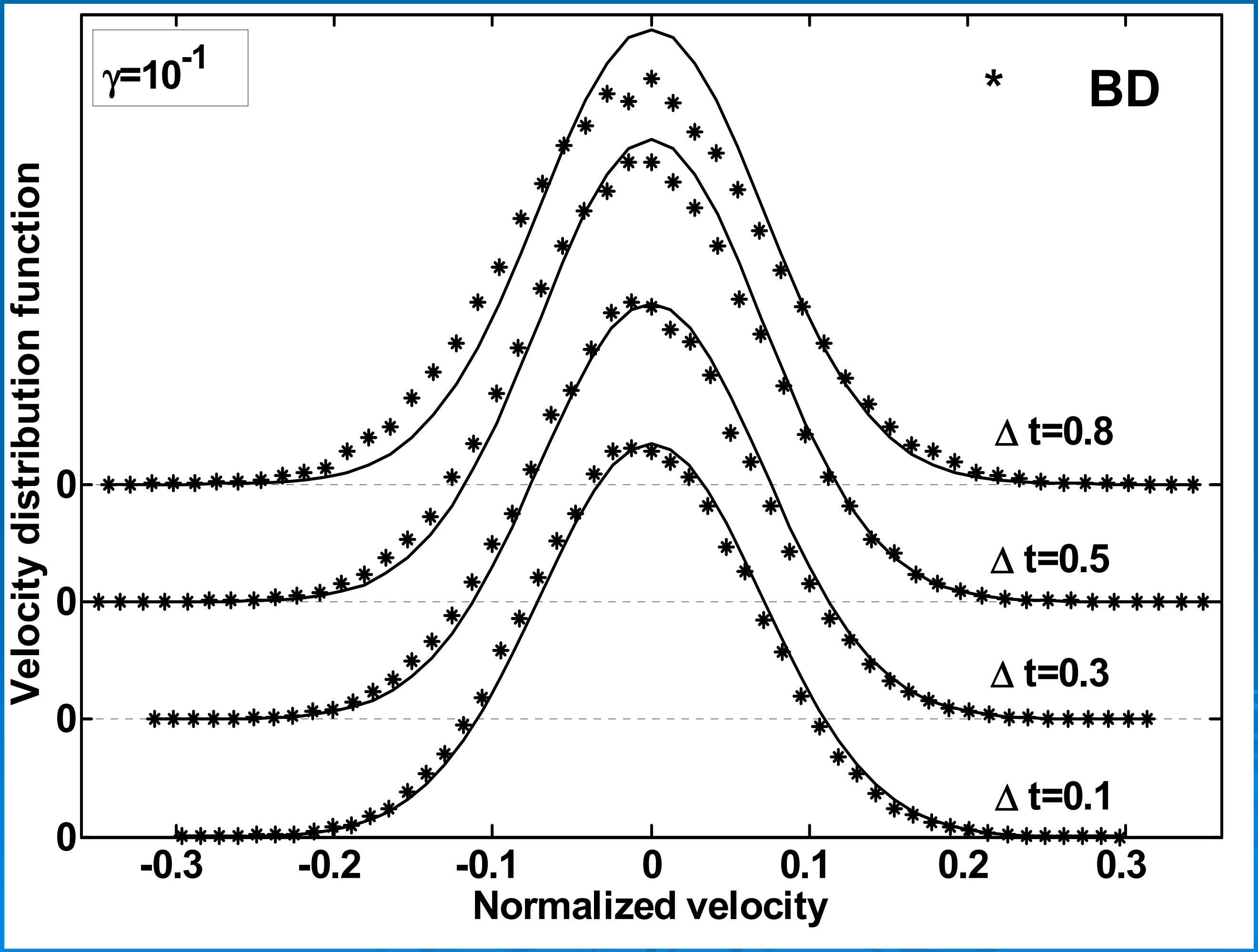
- Equilibrium states
  - Crystal structures (radial distribution function)
  - Phonon spectra
  - Time-correlations
- Non-Equilibrium interactions
  - Excited Mach Waves in the plane
  - Forces on the test particle(s)

Test: energy conservation

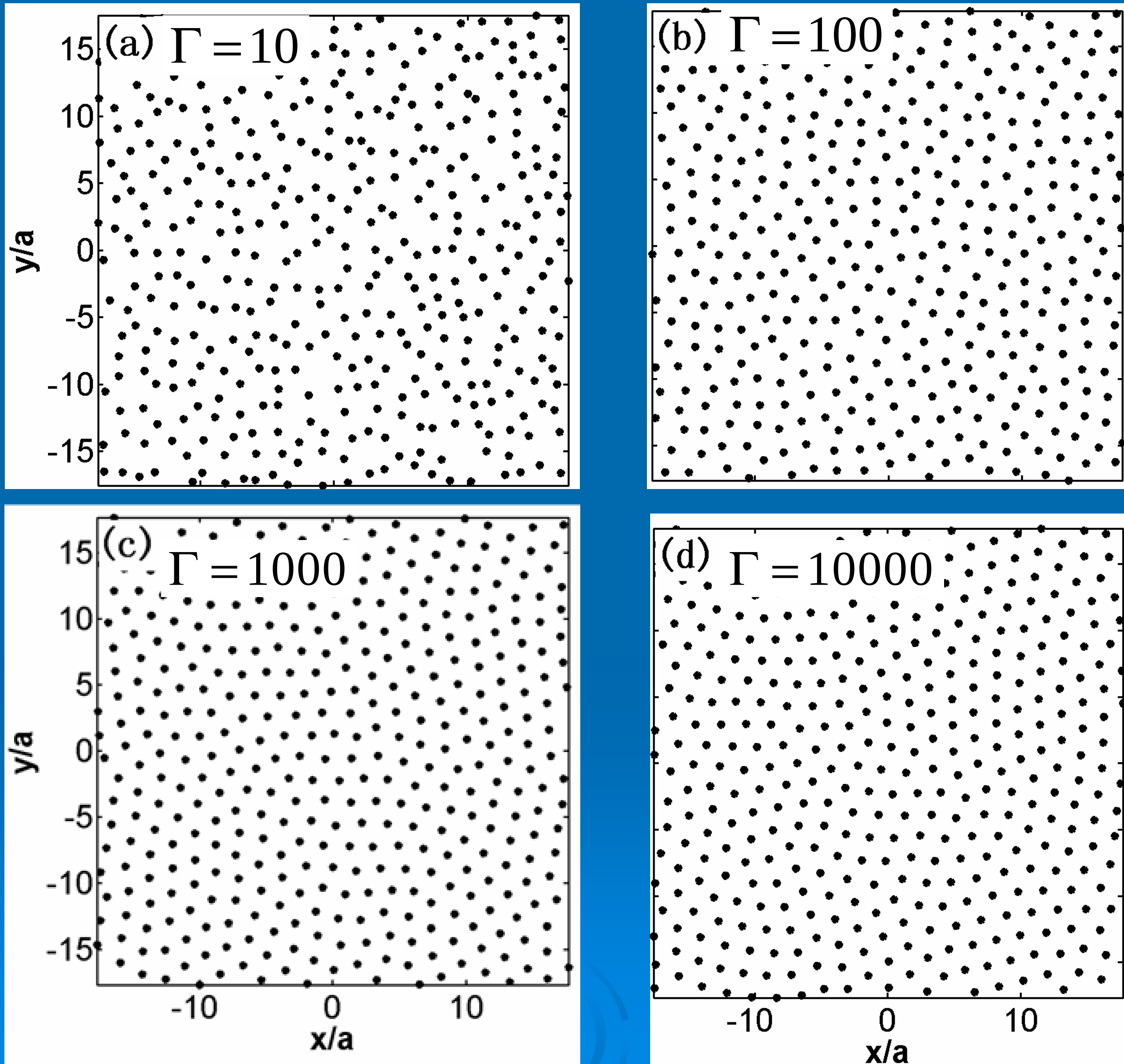


Test: velocity distribution

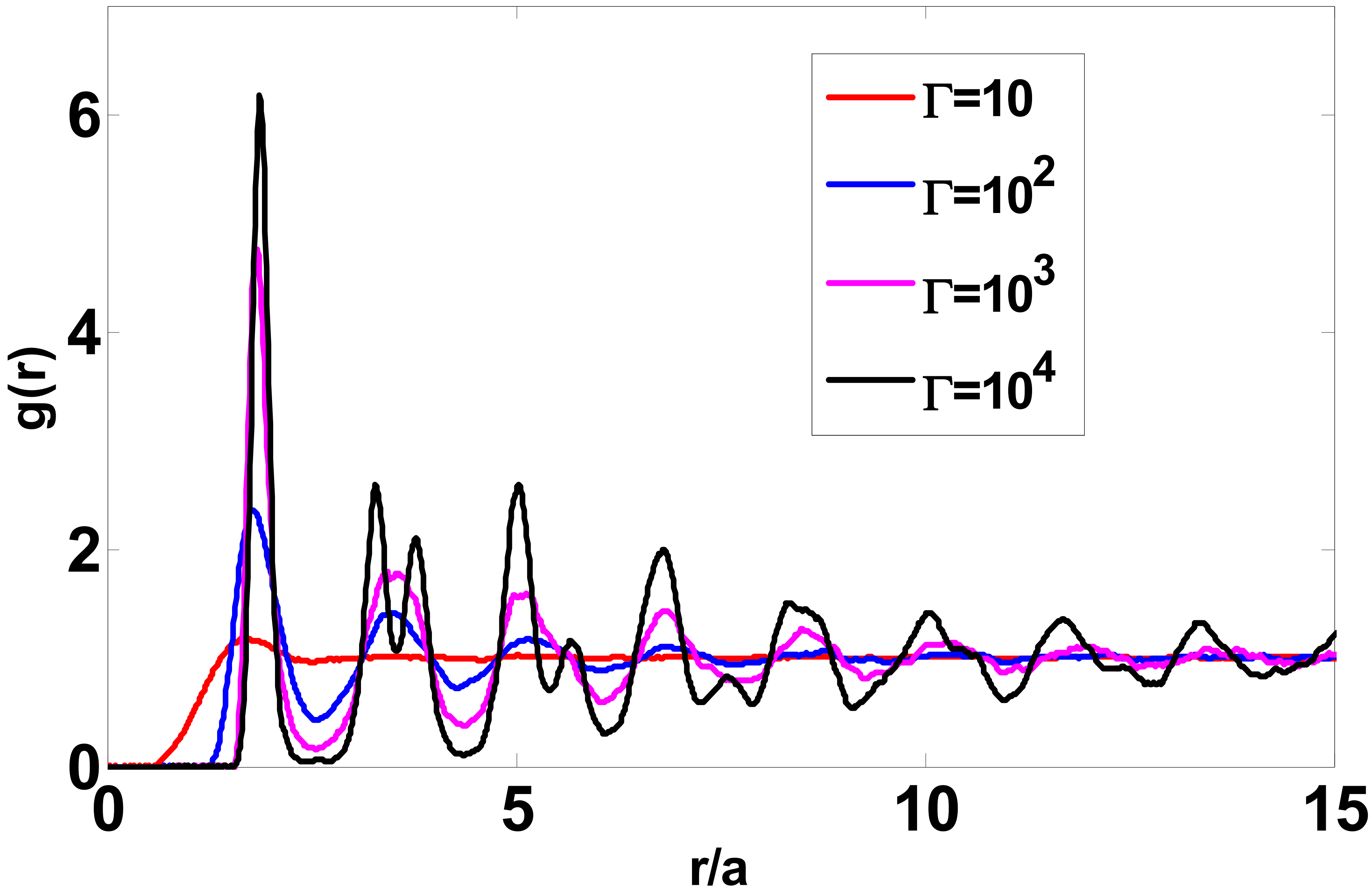
Liquid state with $\Gamma = 360$ and $\kappa = 2$



$\kappa = 1$

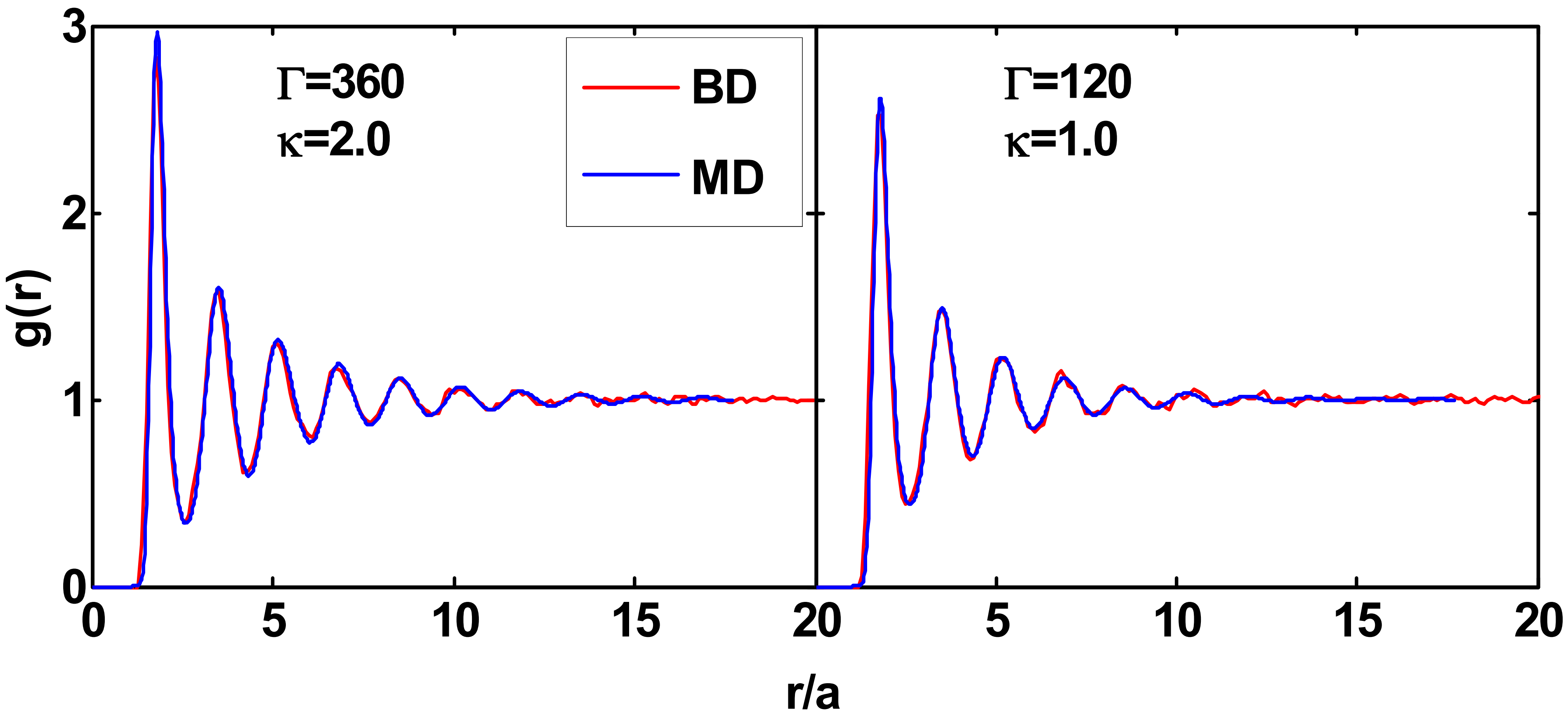


Statics: radial distribution function



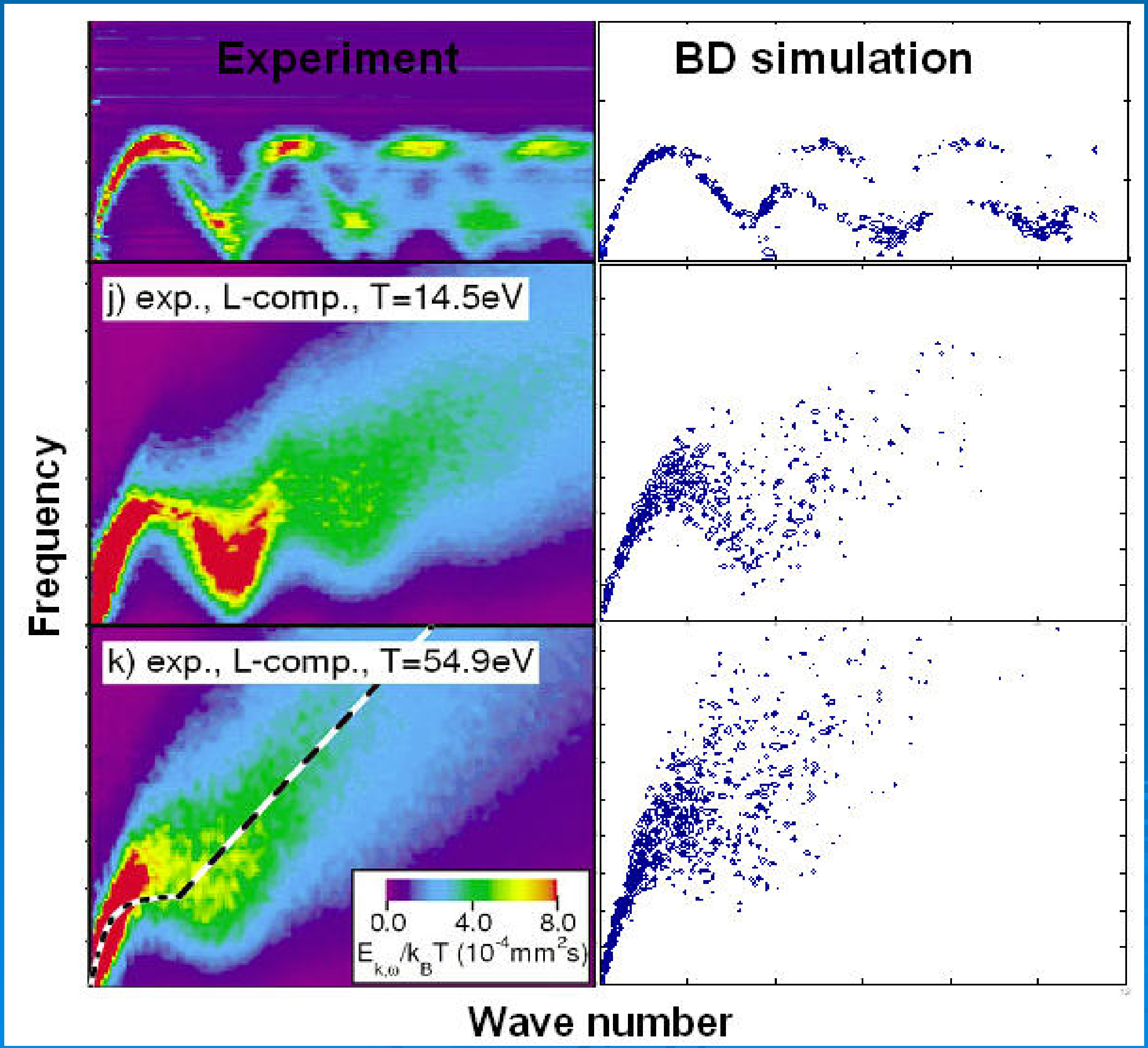
A double check: static structure

Comparison with previous MD simulation

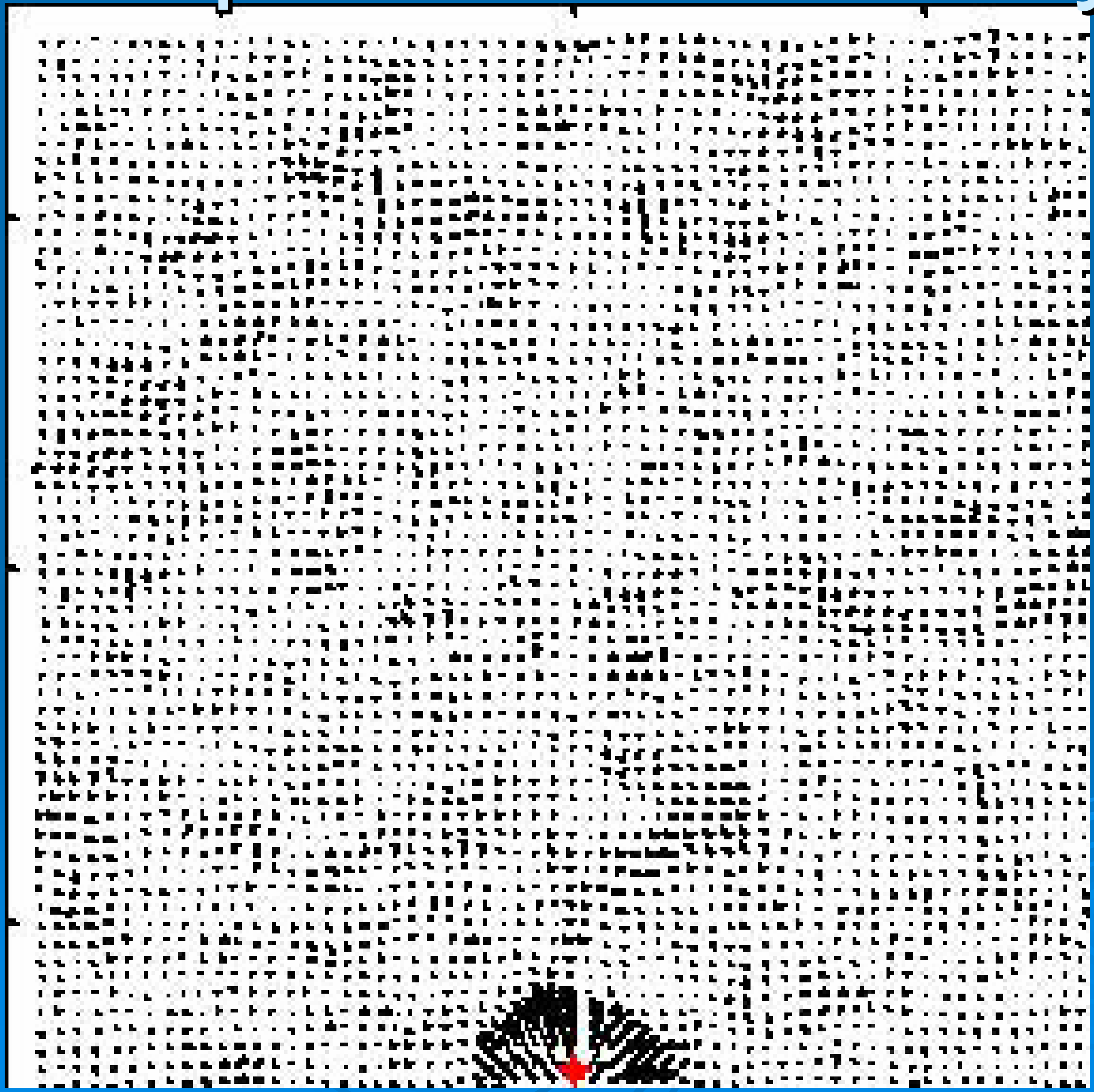


MD data from: Kalman et al., PRL 92, 065001 (2004)

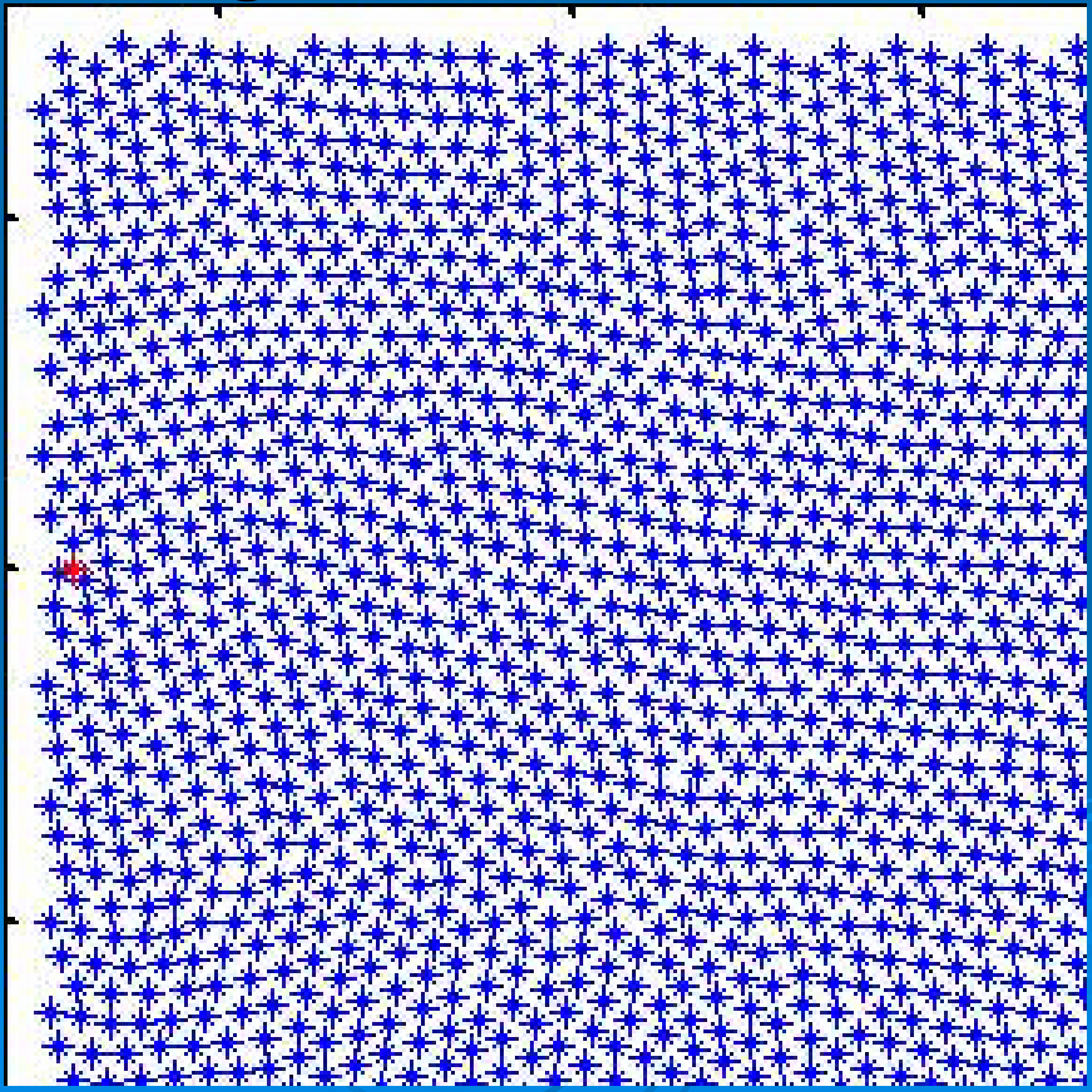
Dynamics: phonon energy spectra



Mach cone excited by external charge: experimental velocity field



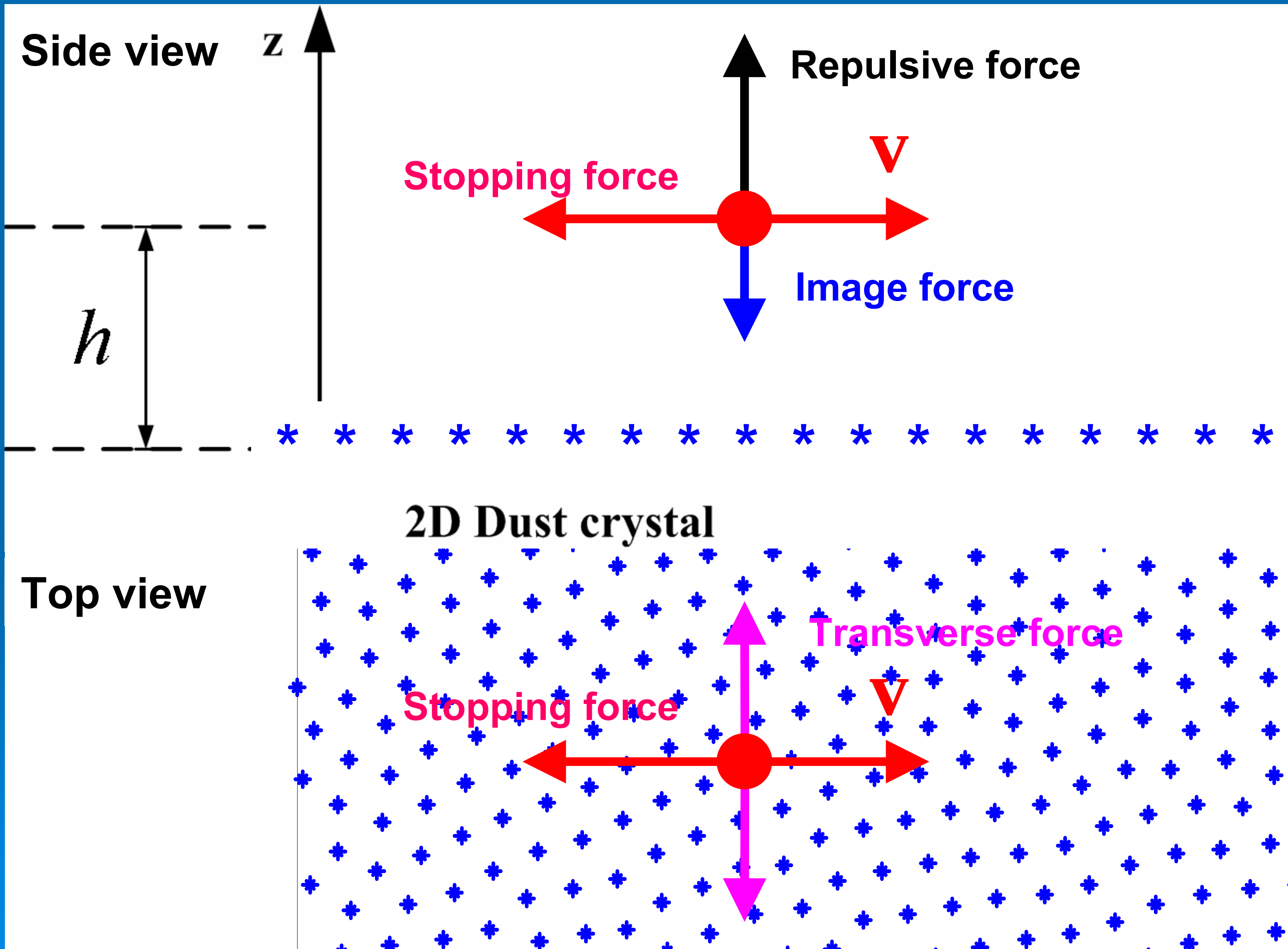
Mach cone excited by external charge: BD simulation



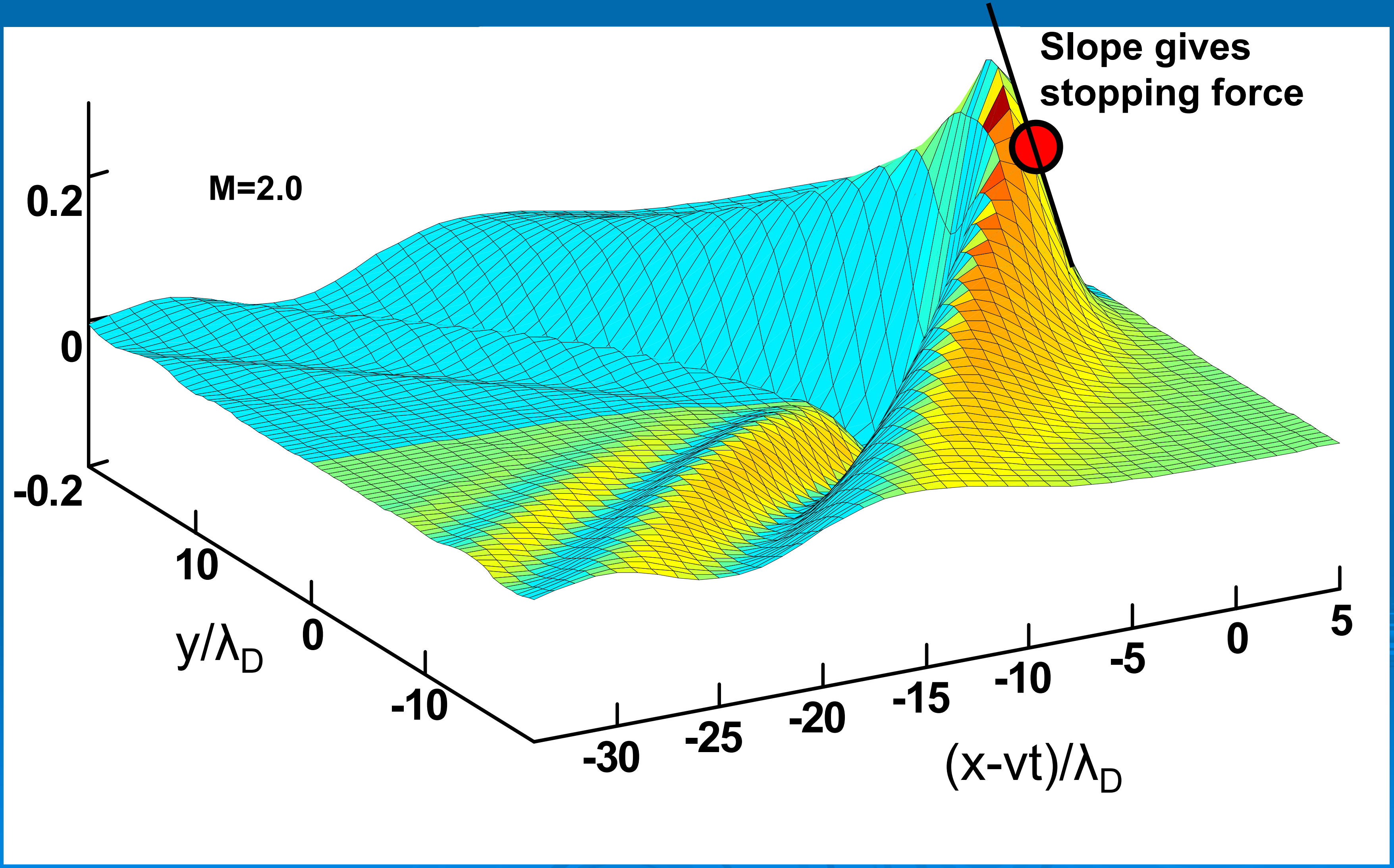
OUTLINE

- Introduction to Dusty Plasma Physics
- Structures and Waves in Dust Layers
- Mach Cones in Dust Layers
  - Excited by: moving laser & external particle
  - Experiment, analytical models & simulation
- Polarization Forces on External Particle
- Details of Modeling and Simulation

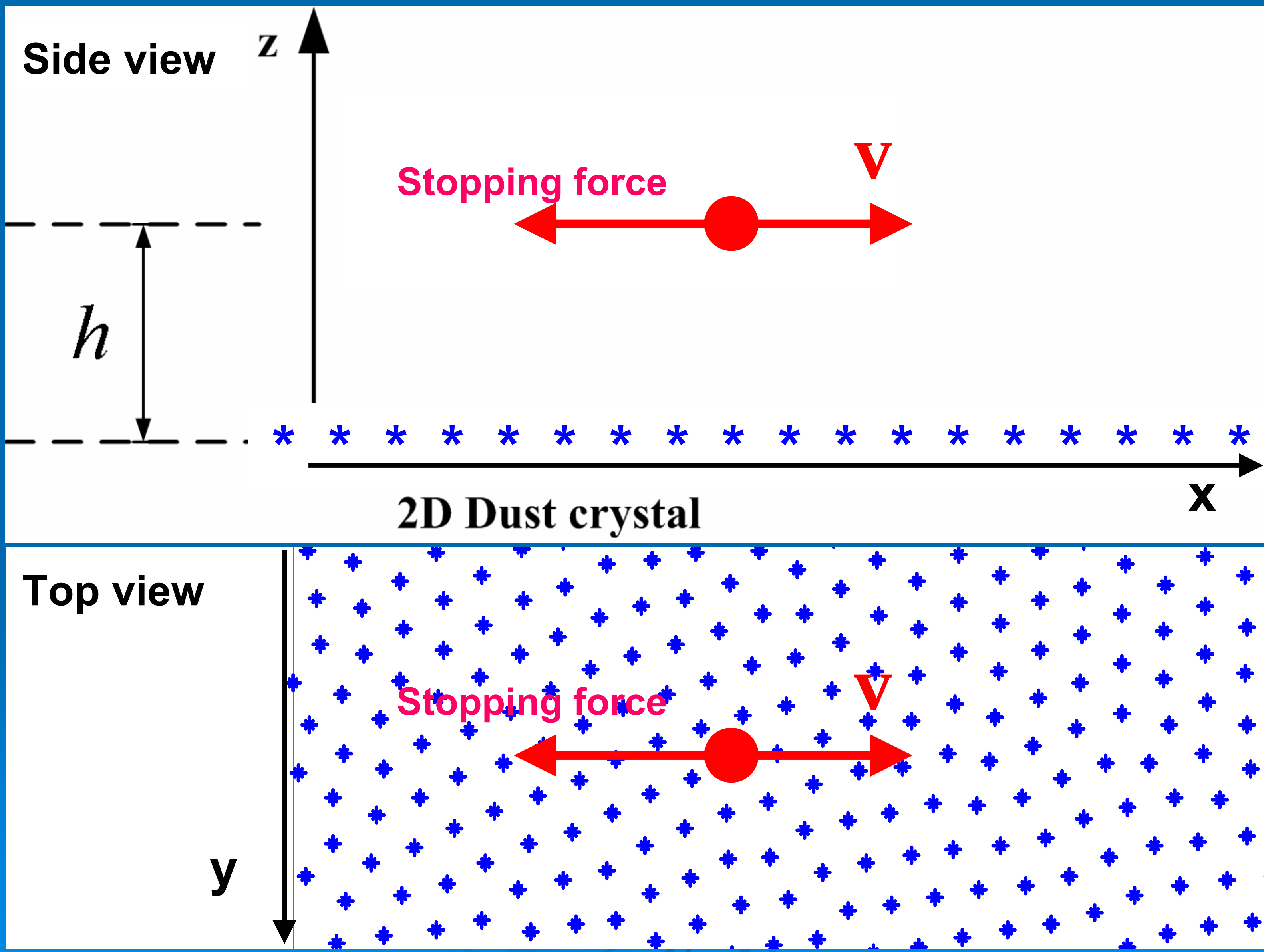
Forces on the moving particle



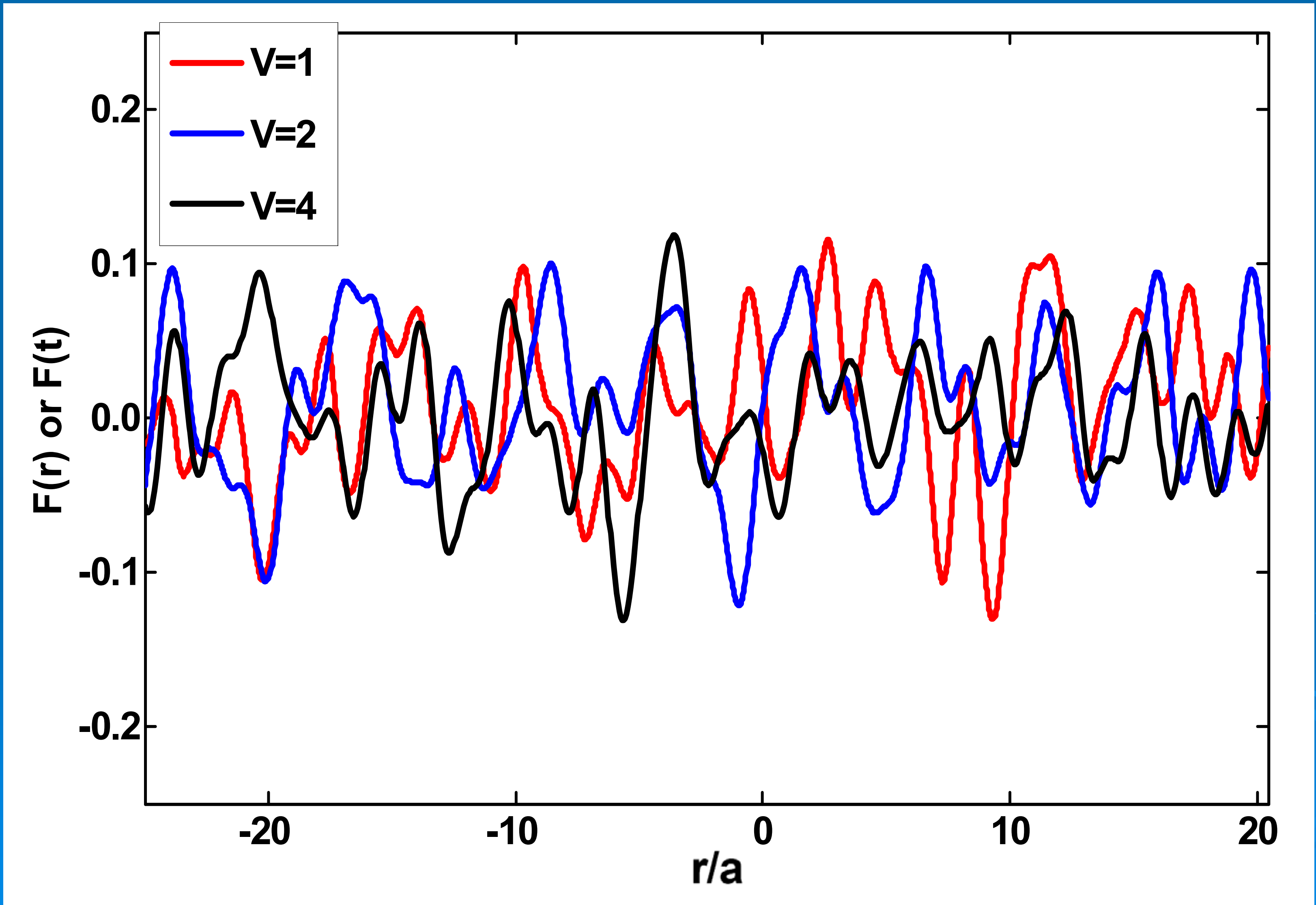
Induced potential (hydrodynamic model)



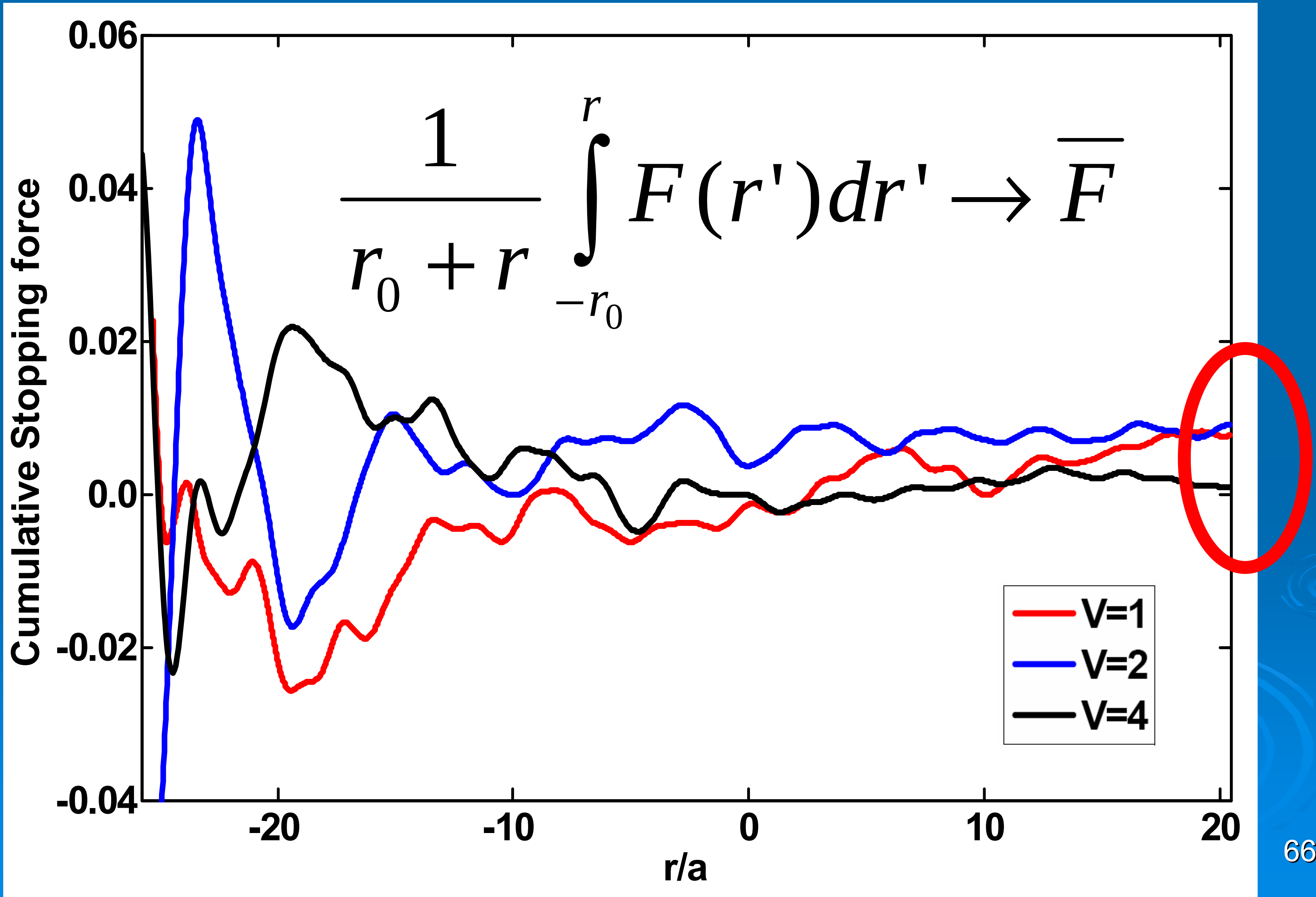
Stopping force on the moving particle



Position dependent stopping force from BD sim.

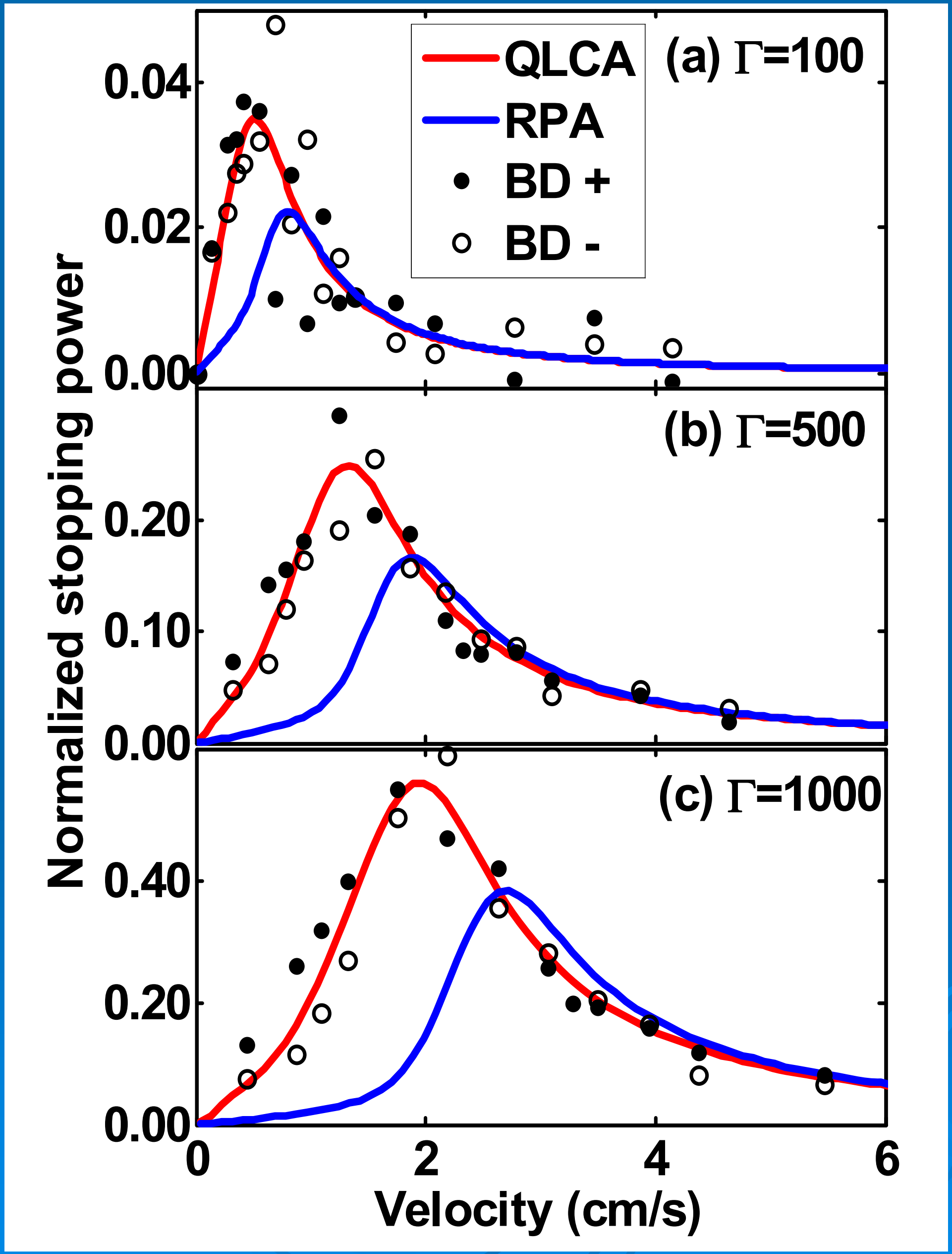


Evaluating the mean stopping force from the cumulative force

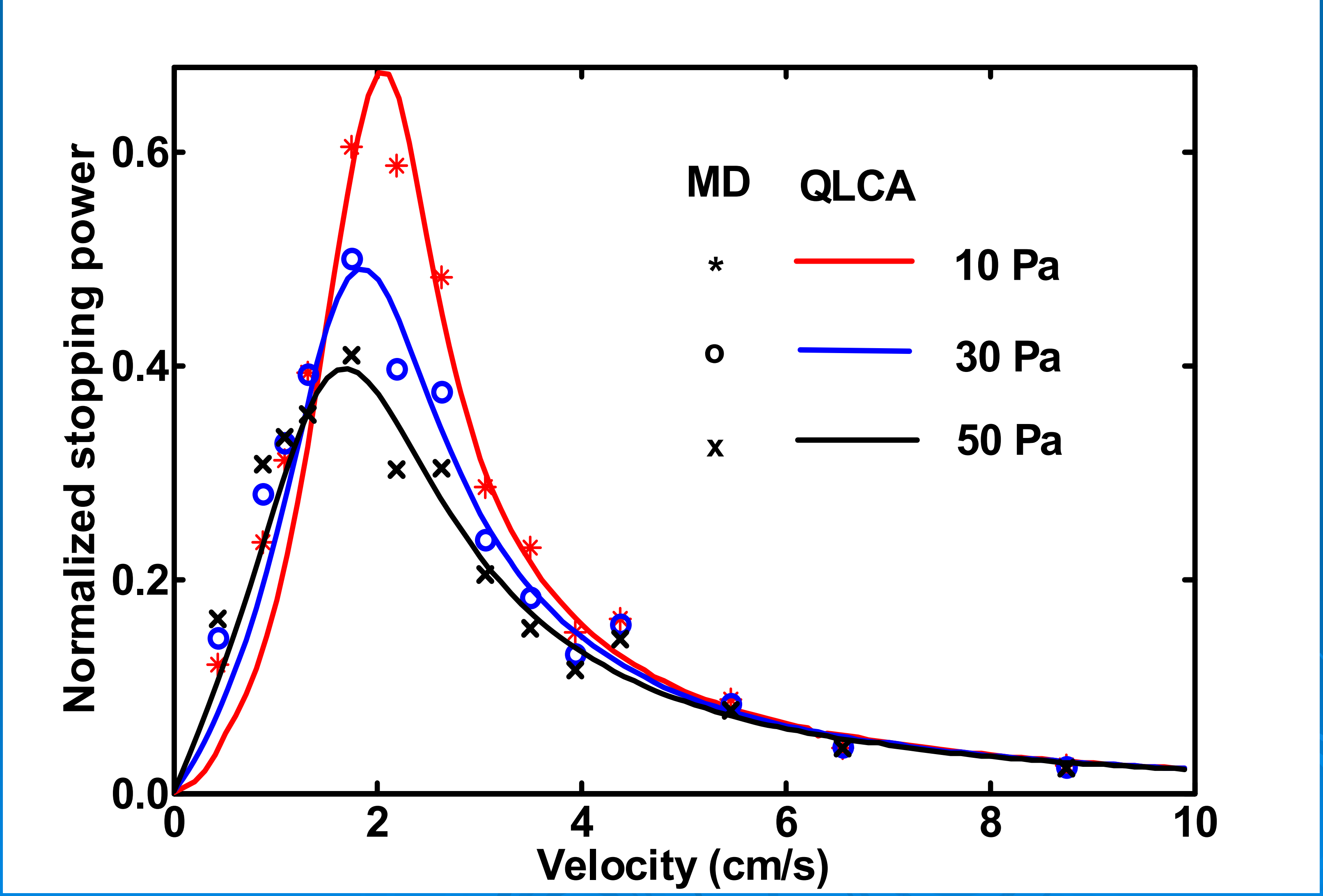


$$\frac{1}{r_0 + r}\int_{-r_0}^{r} F(r')dr' \rightarrow \overline{F}$$

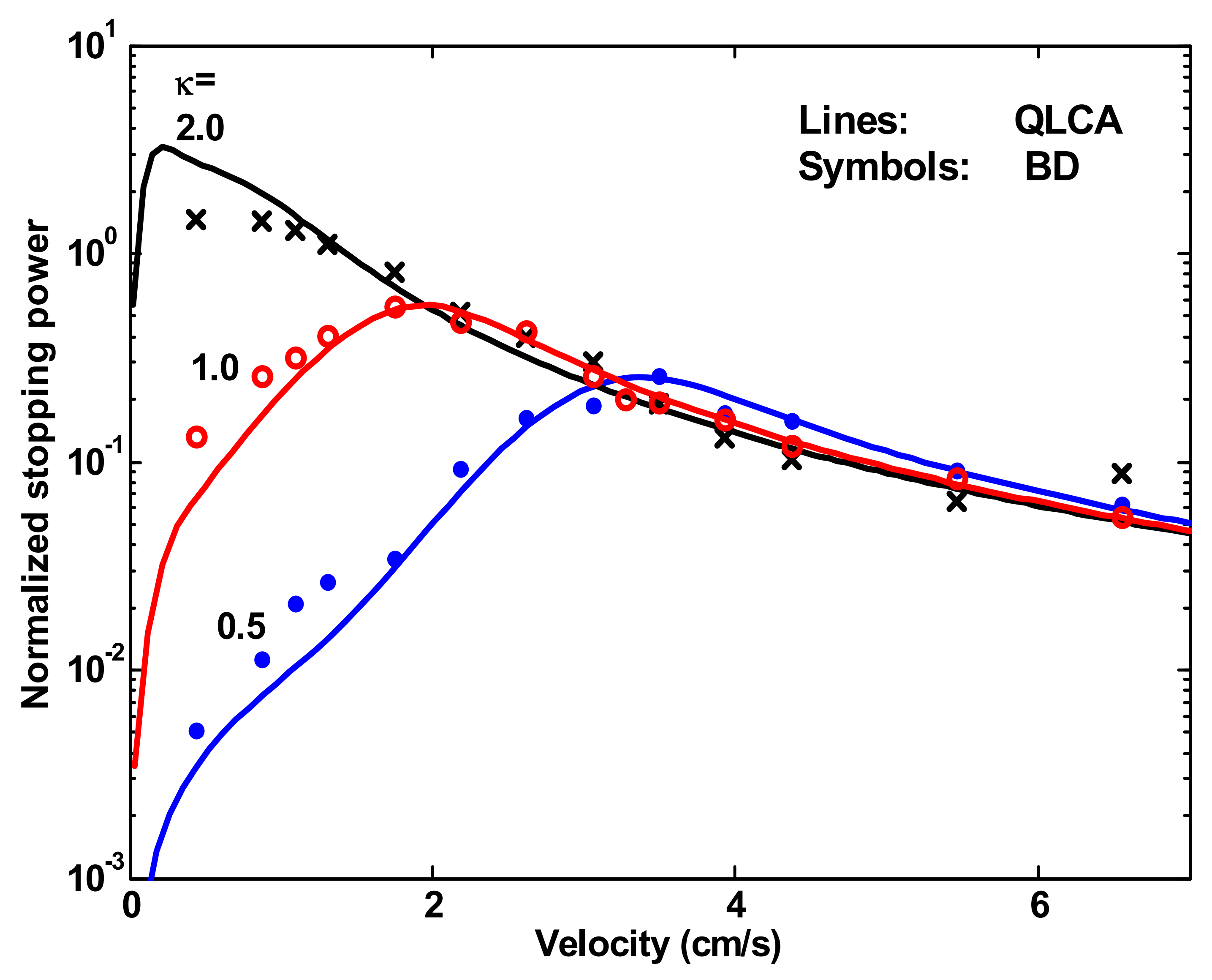
Mean stopping force



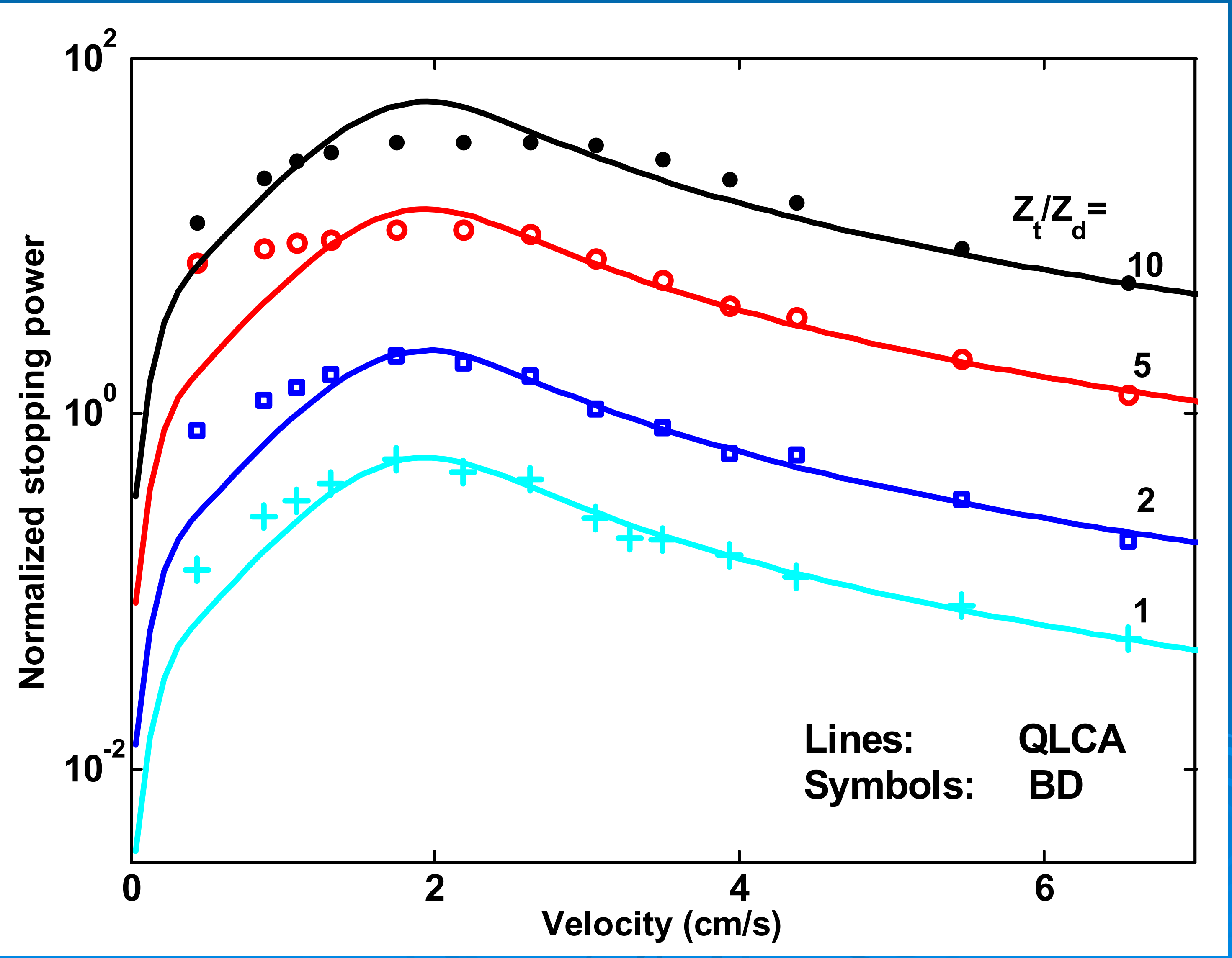
Mean stopping force



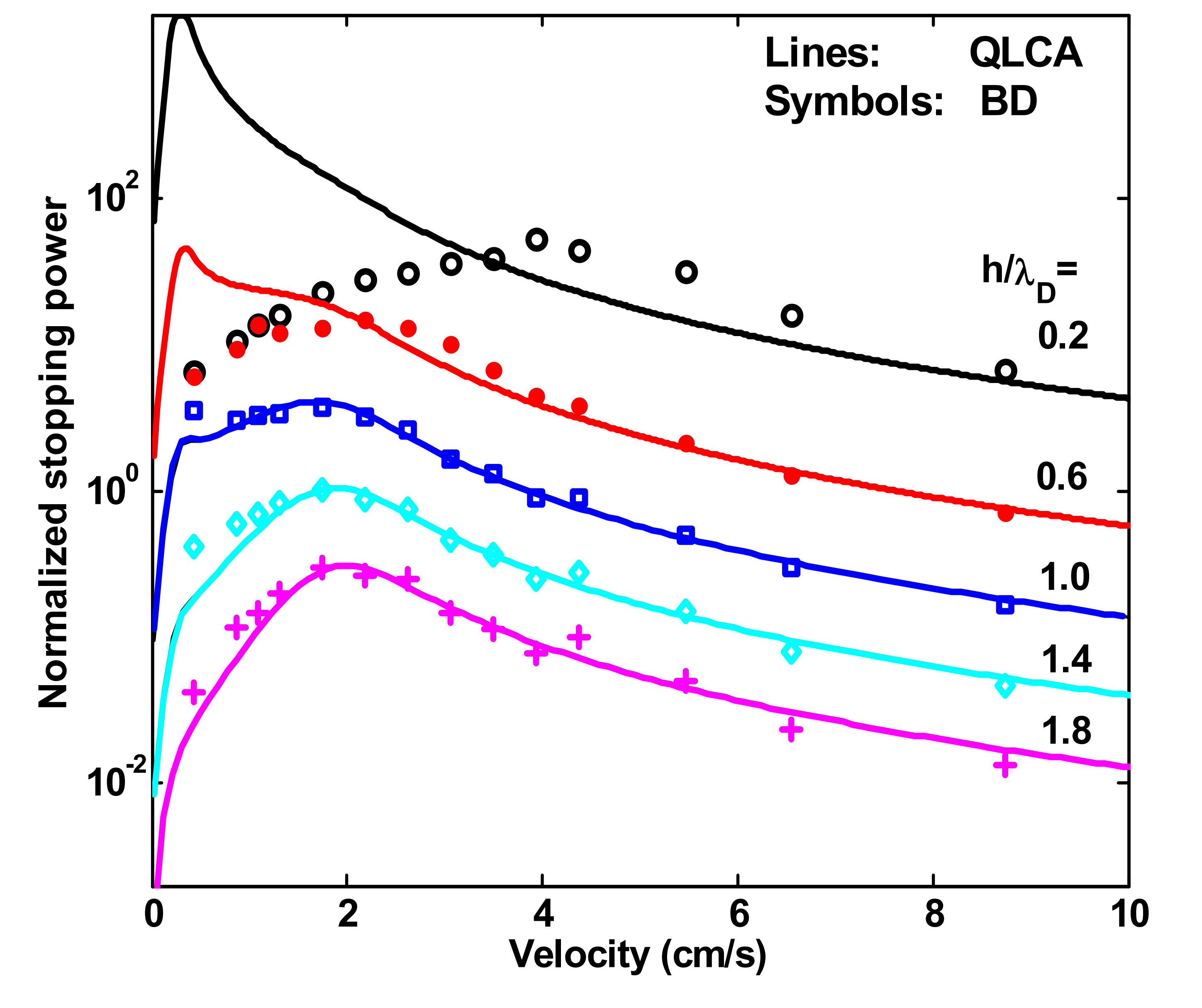
Mean stopping force



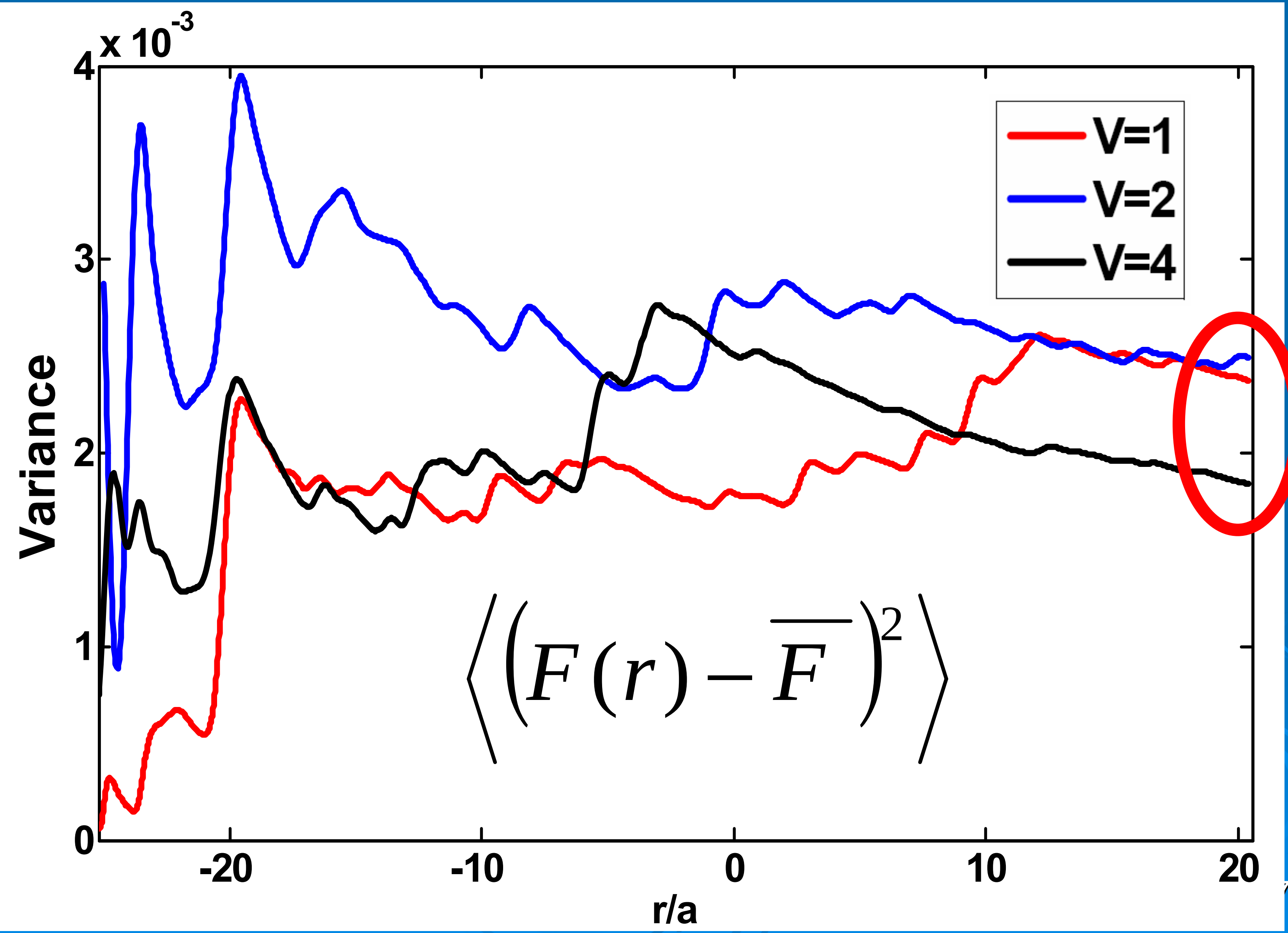
Mean stopping force



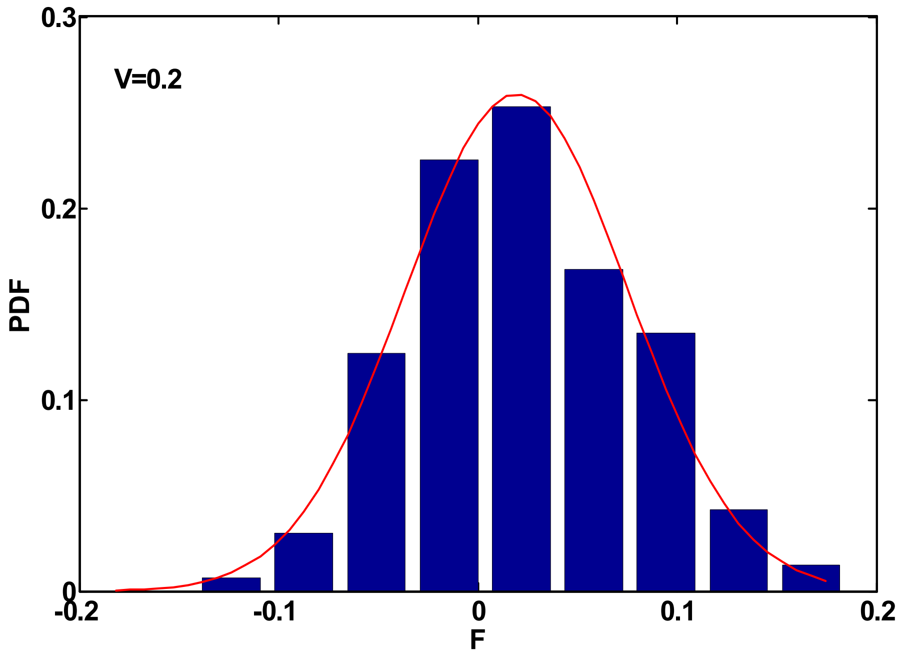
Mean stopping force



Variance of stopping force

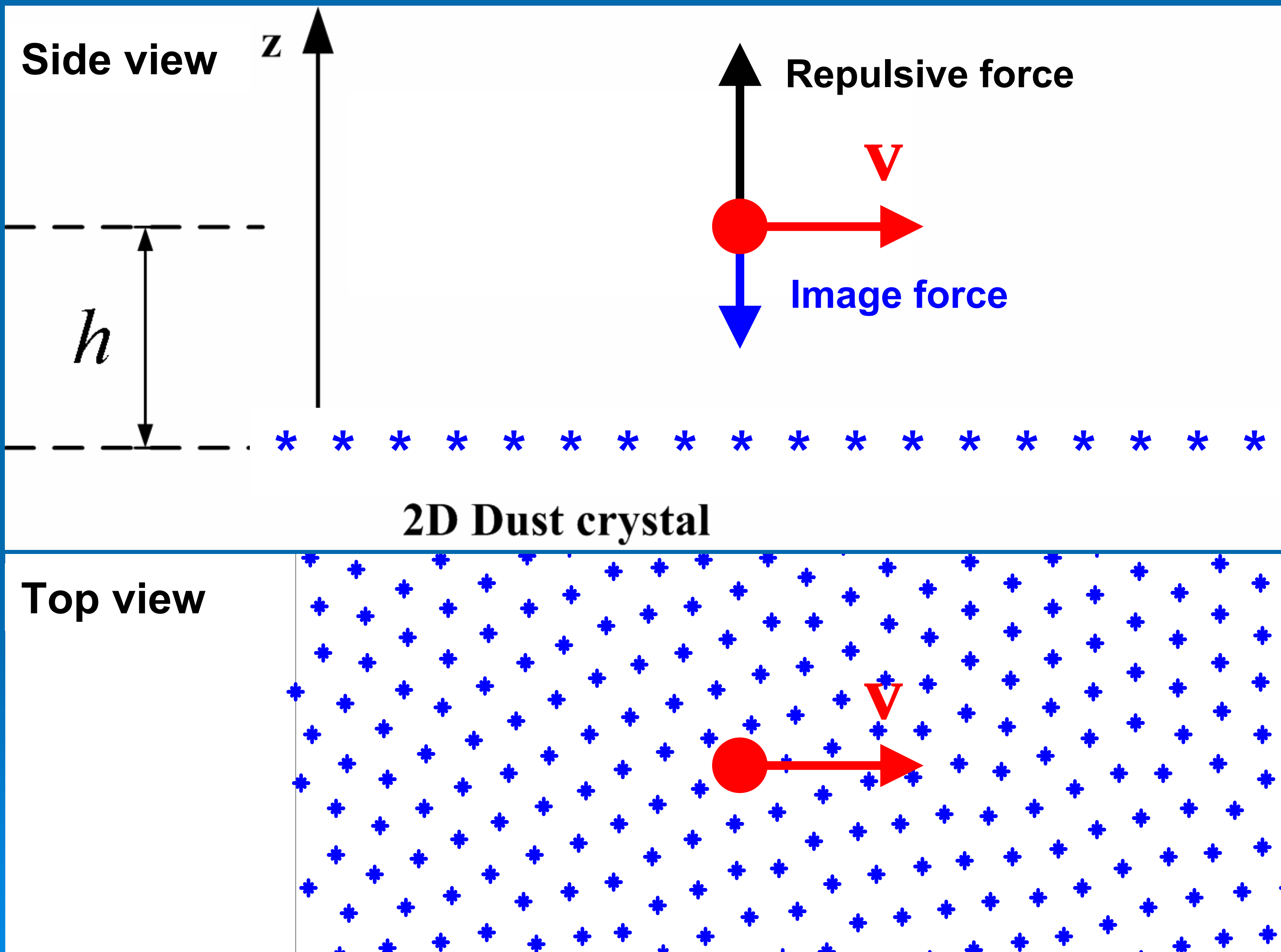


Probability distribution of stopping force - straggling

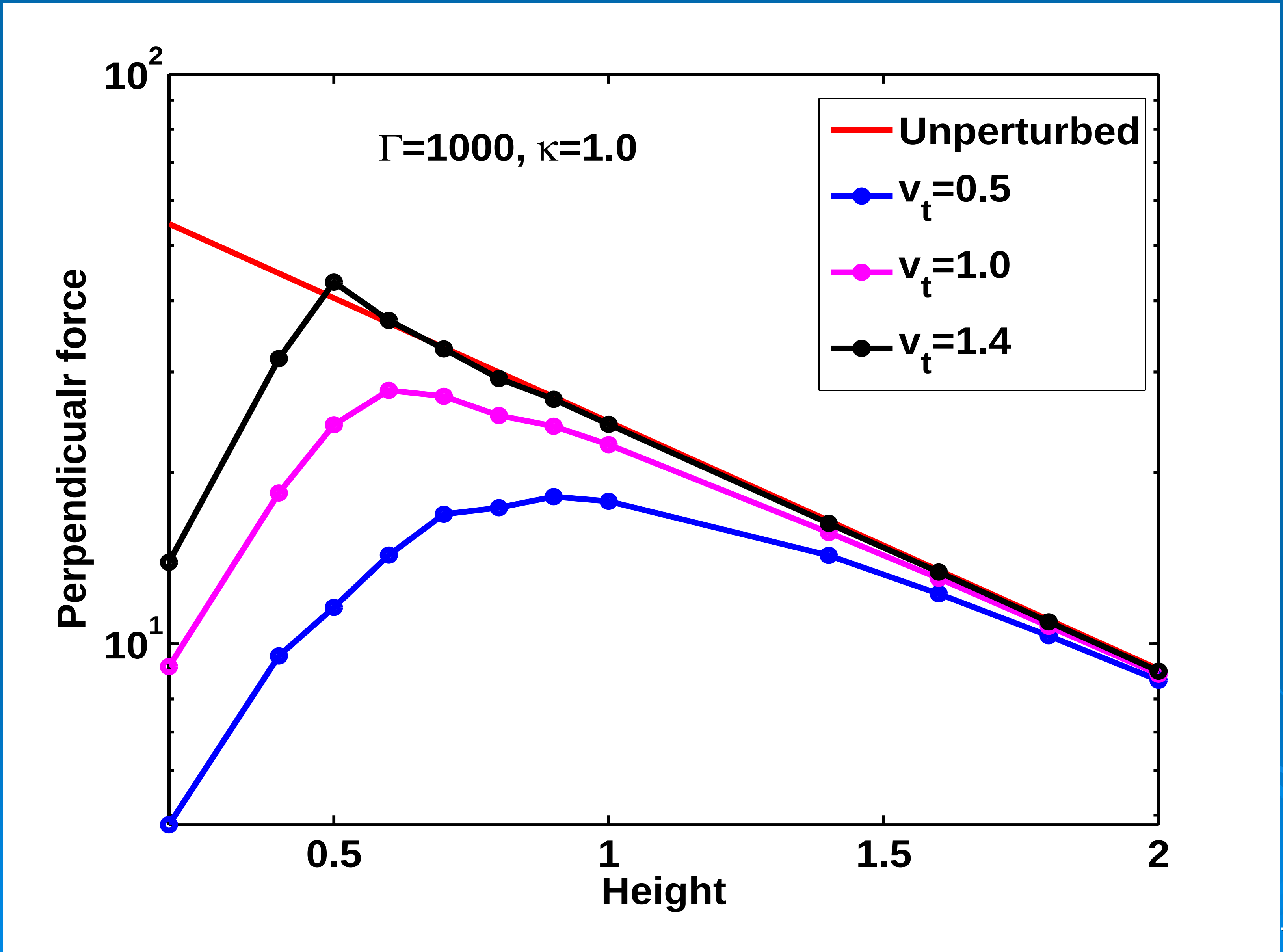


Bars : direct measurements of simulation data red lines: Gaussian distributions with the above-calculated means and variances.

BD sim. of the image force



Total perpendicular force



Repulsive (unperturbed) force

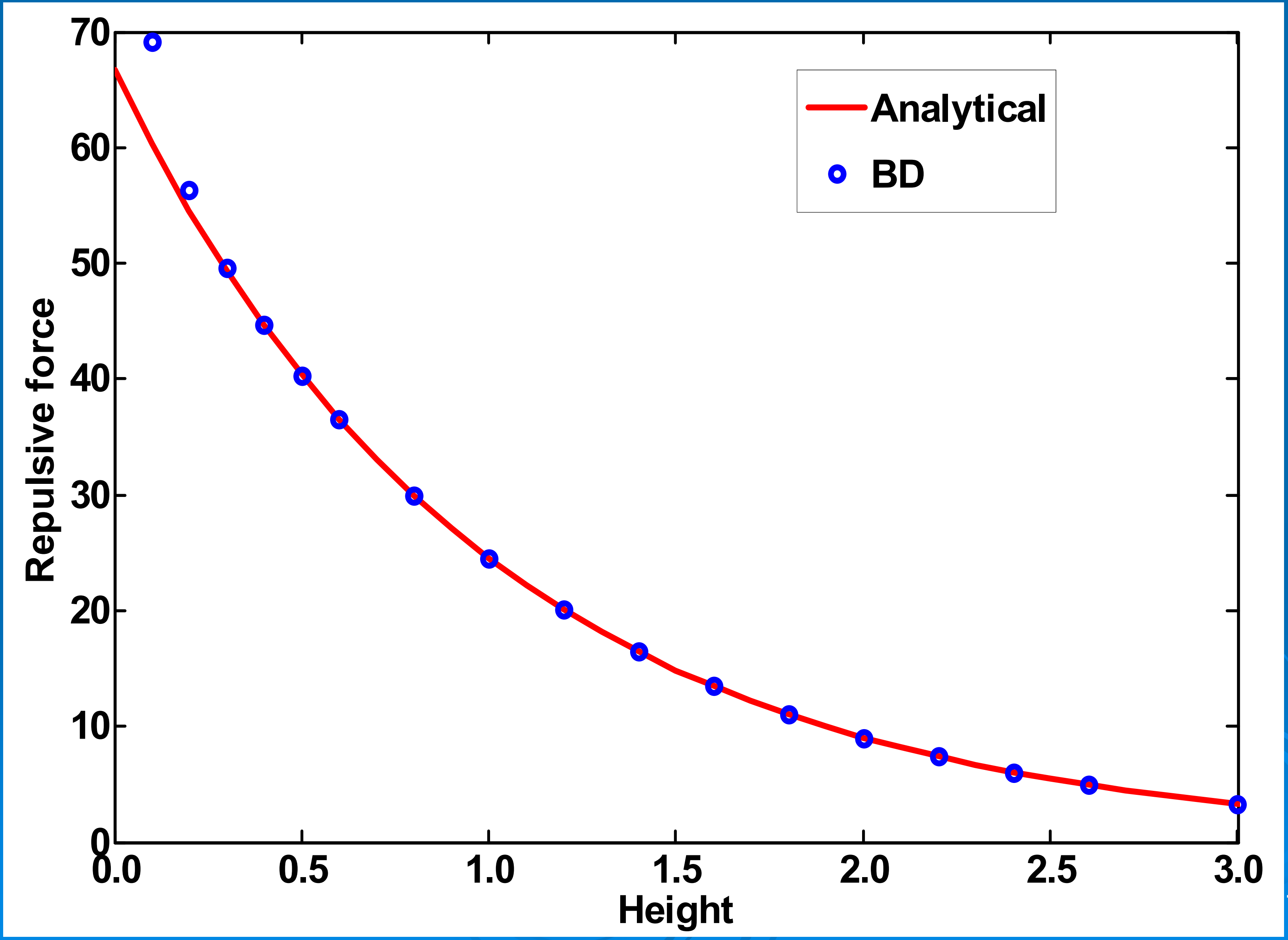


Image force: h=2.0

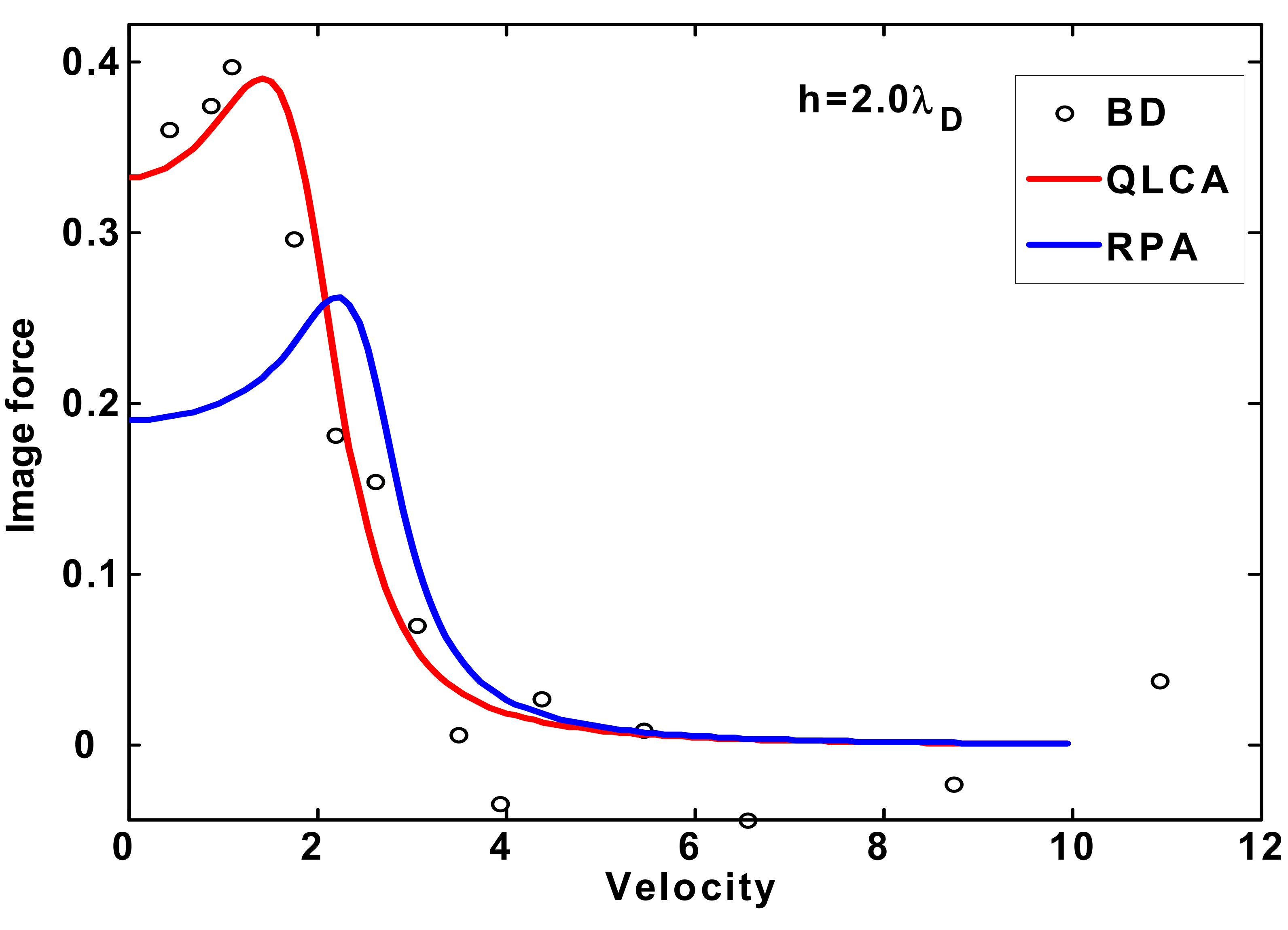


Image force: h=1.0

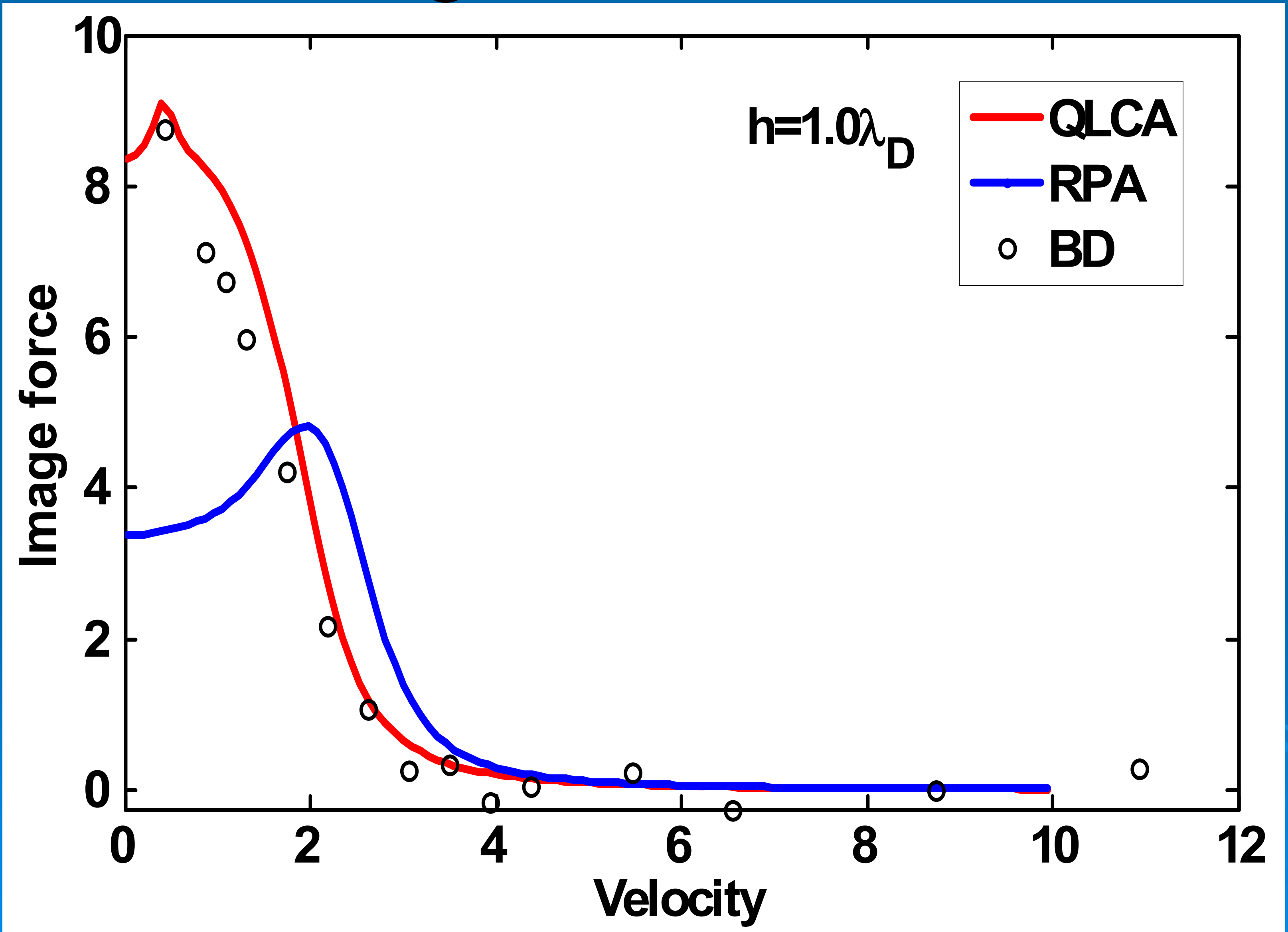


Image force: h=0.5

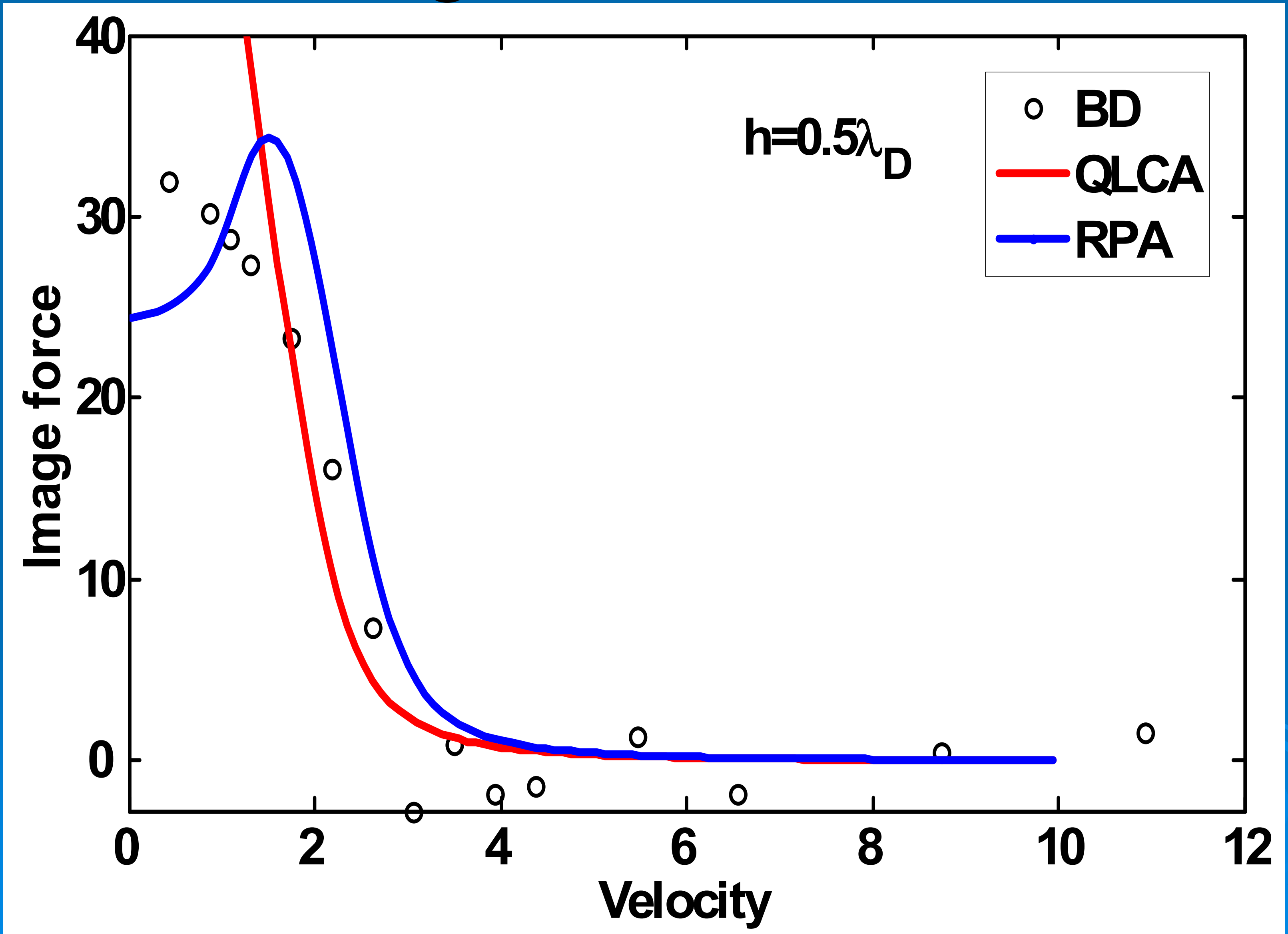
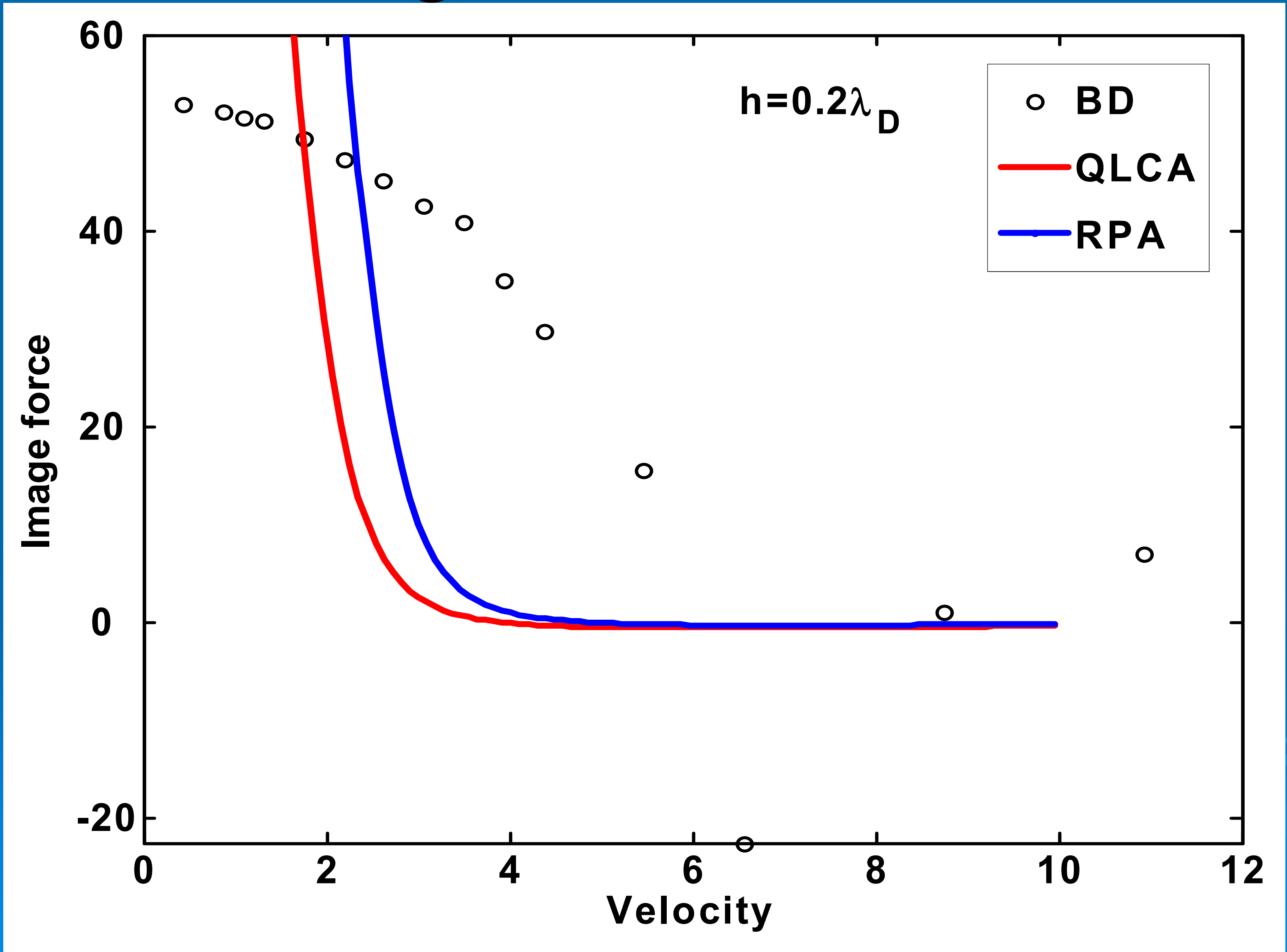


Image force: h=0.2



Projectile-target coupling strength

$$\Theta(h,r,v) = \frac{V_{td}}{V_{dd} + m_d v^2 / 2}$$

$$\max\{V_{td}\} = \frac{|Q_t Q_d|}{h} \exp(-\frac{h}{\lambda_D})$$

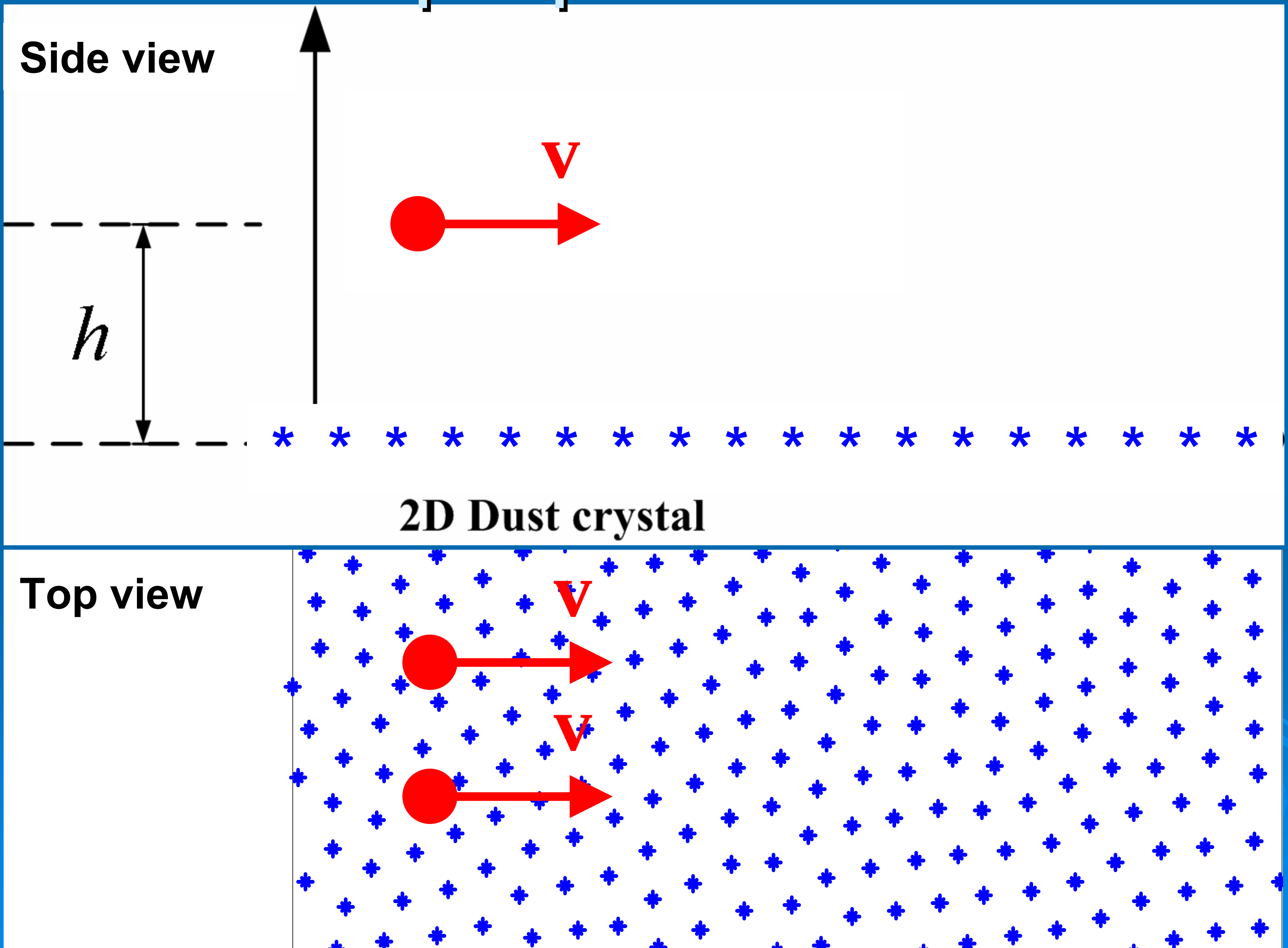
$$V_{dd} = \frac{Q_d^2}{a} \exp(-\kappa)$$

Θ « 1 criterion for validity of linear theory (QLCA & RPA)

Conclusions

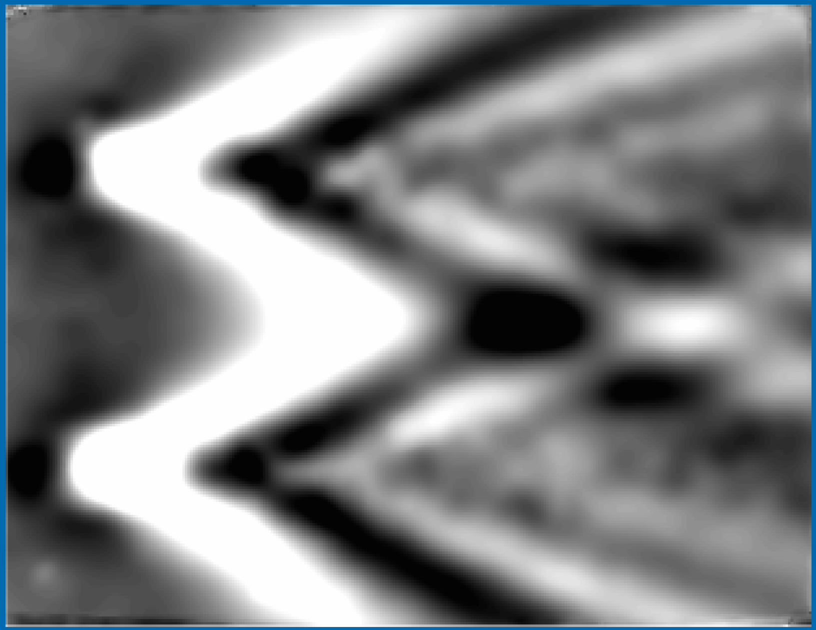
- **Strong-coupling effects** described well by QLCA but RPA fails except at high speeds
- **Non-linear effects** in the projectile-target interactions not described by QLCA & RPA

Vicinage effect for two particles due to superposition of wakes



Experimental vicinage effect

Experimental vicinage effect



Experimental image by Nosenko *et al.*

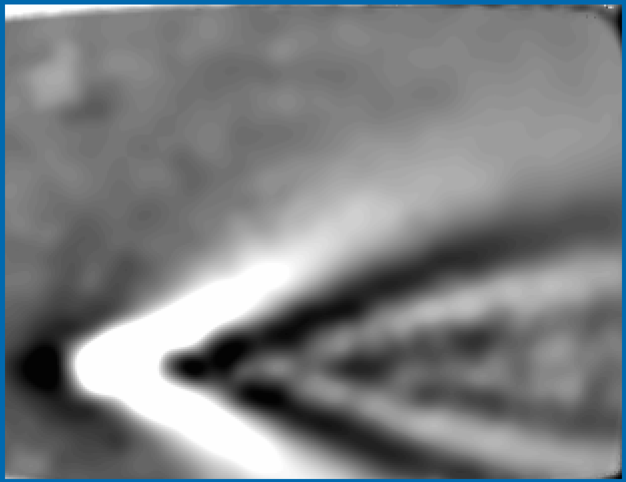
Produced by *two* laser spots moving parallel to each other over dust layer

Credit: Nosenko *et al.*

Test of Linear Superposition

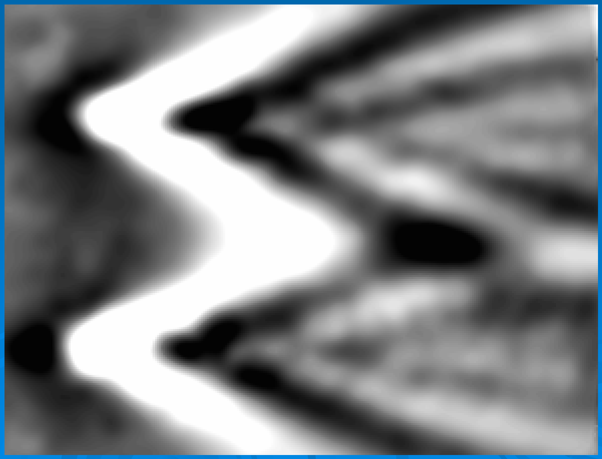
Experimental image 1

+



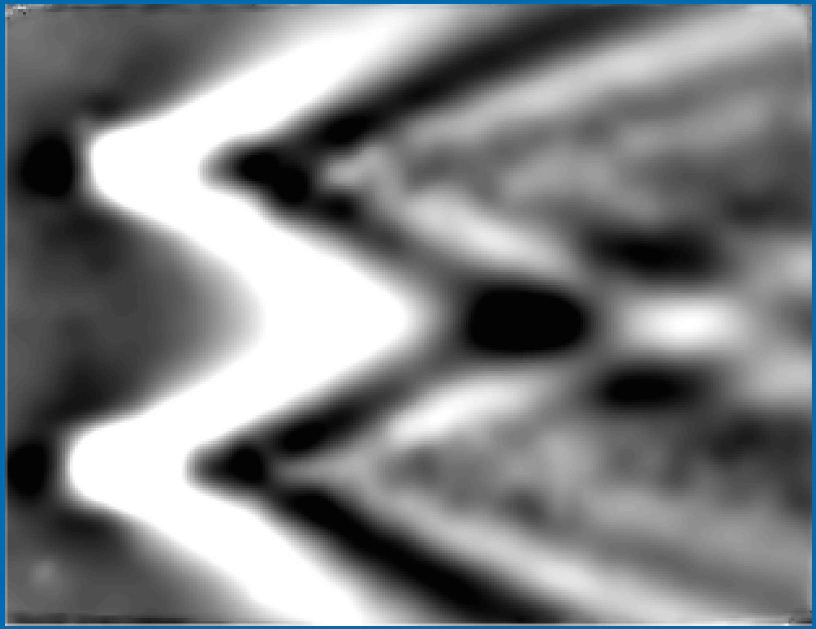
Experimental image 2

=

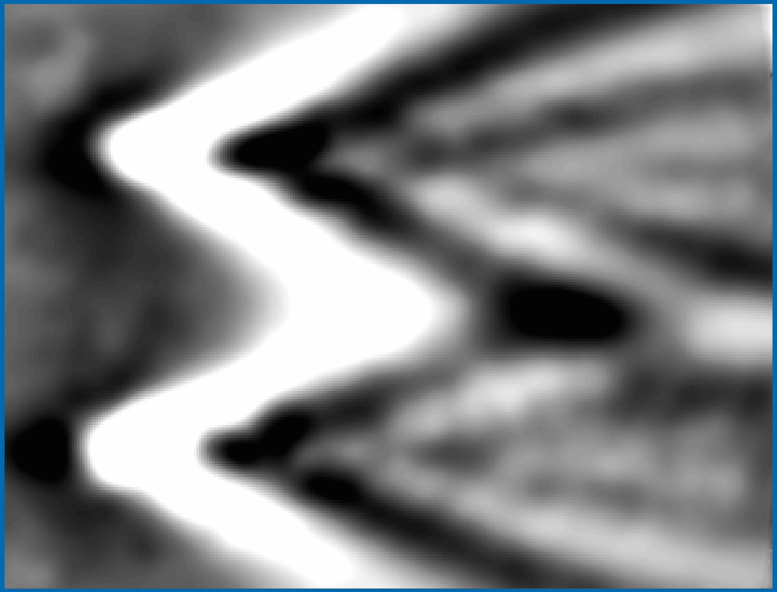


Synthesized image

Test of Linear Superposition



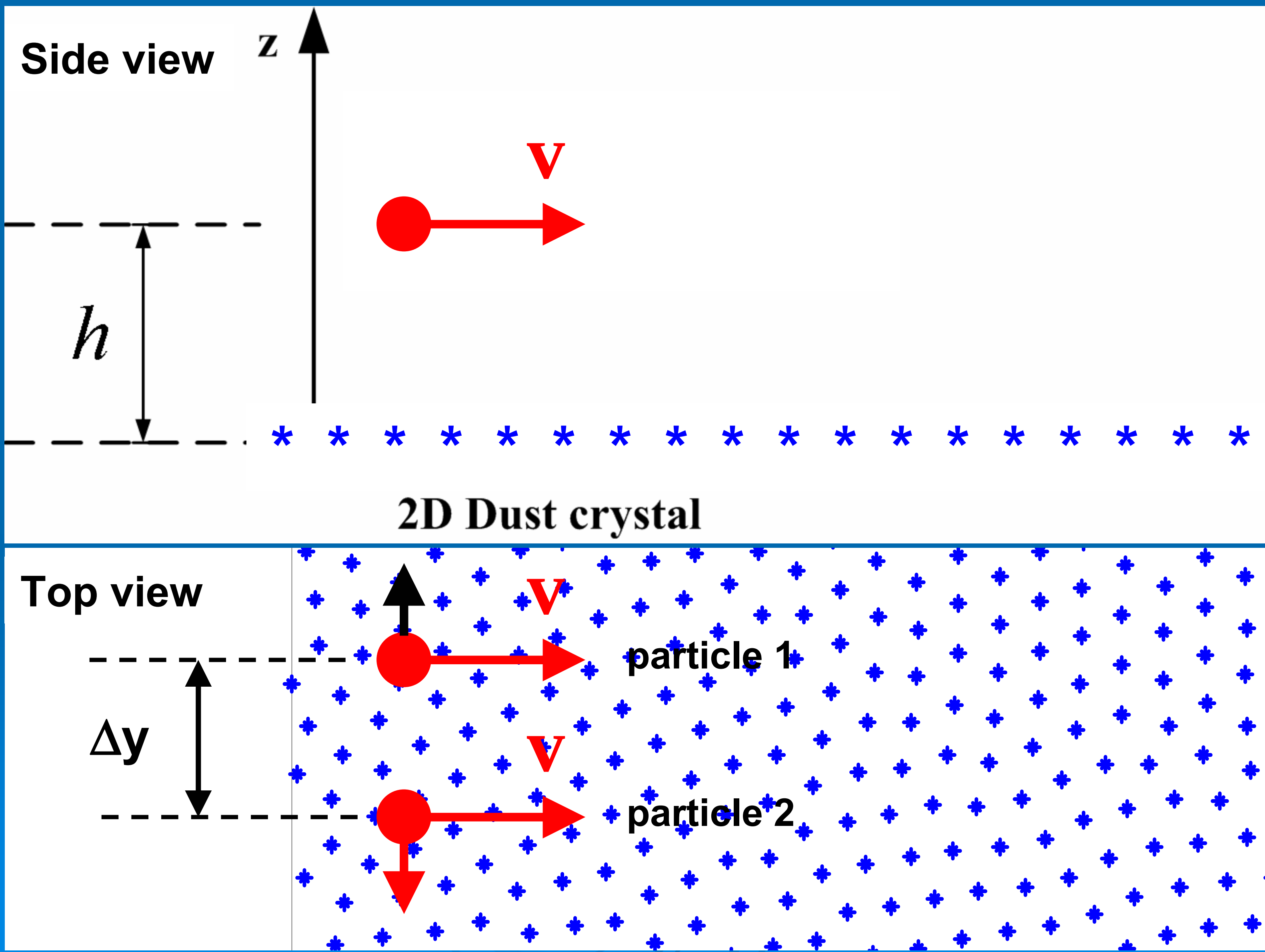
Experimental image



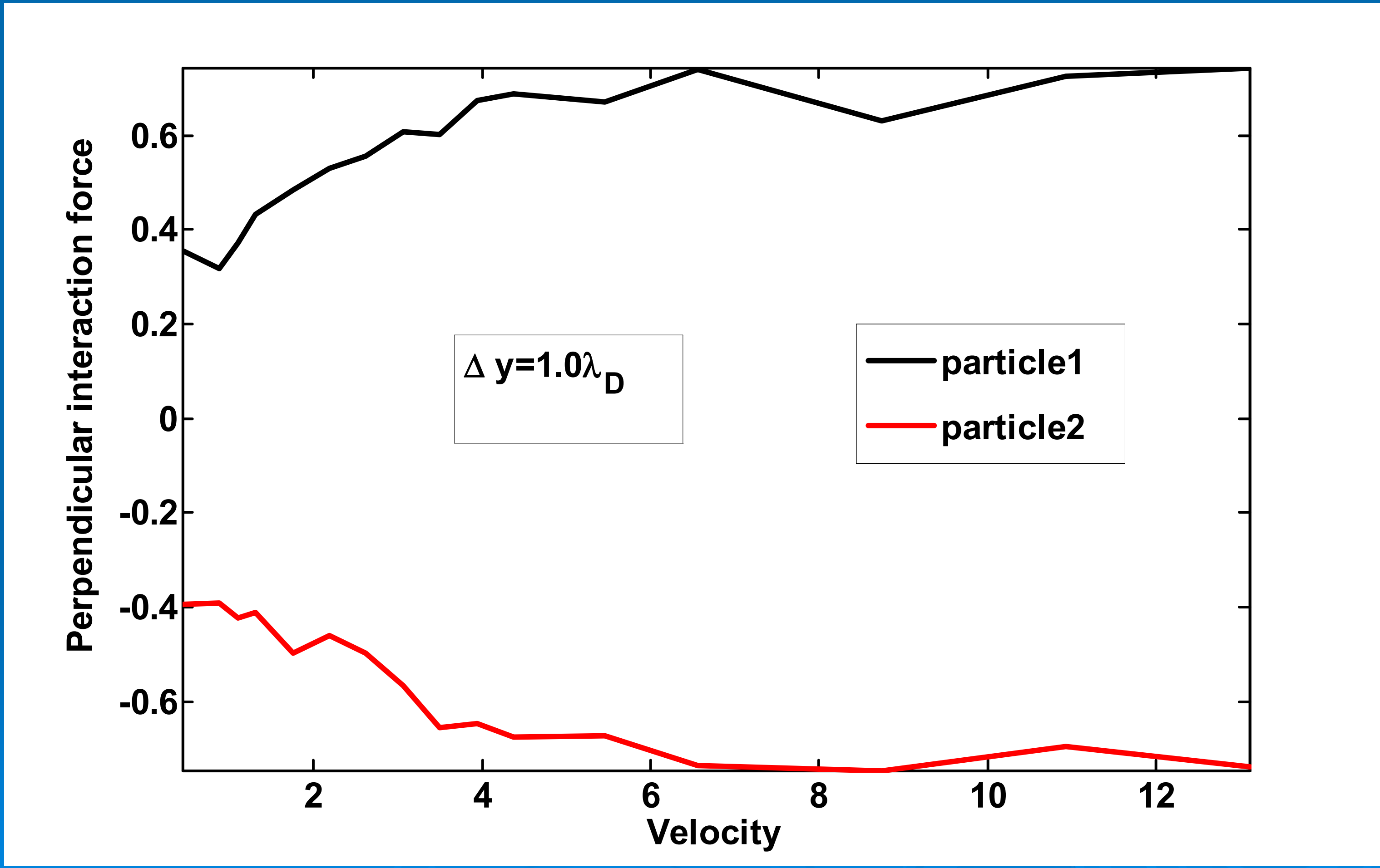
Synthesized image

Agreement ⇒ linear superposition is true

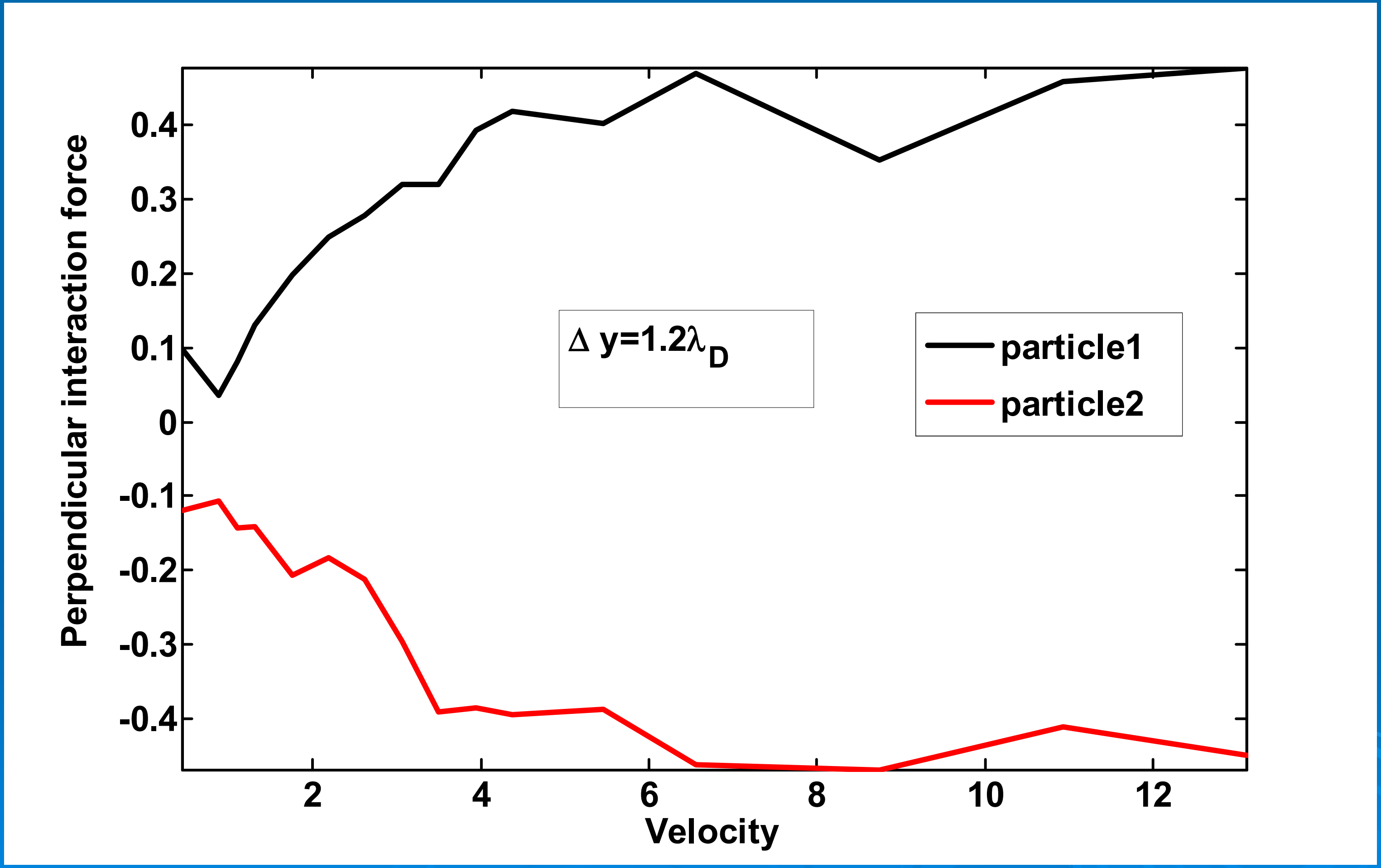
BD sim. of transversal forces



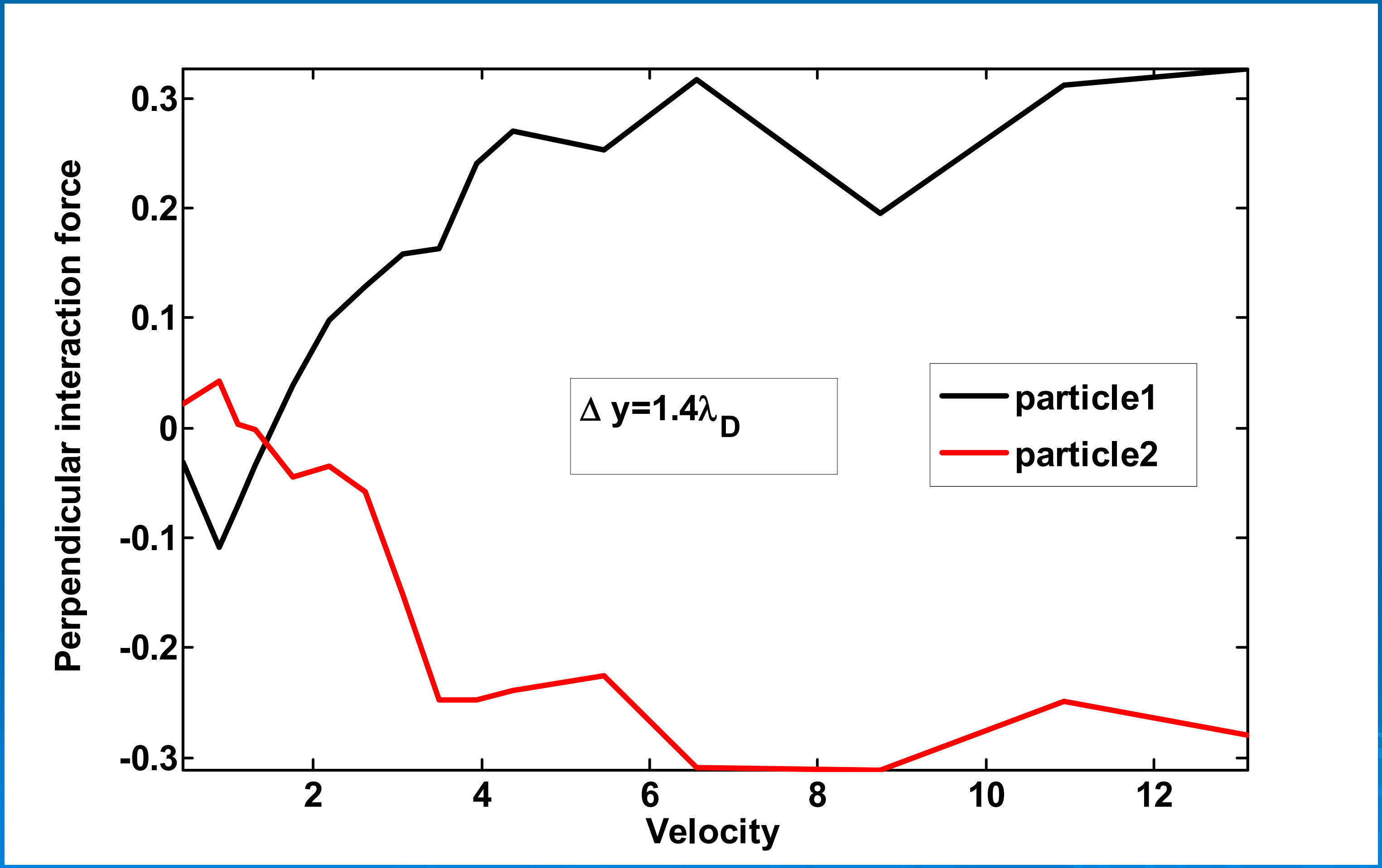
Transversal forces



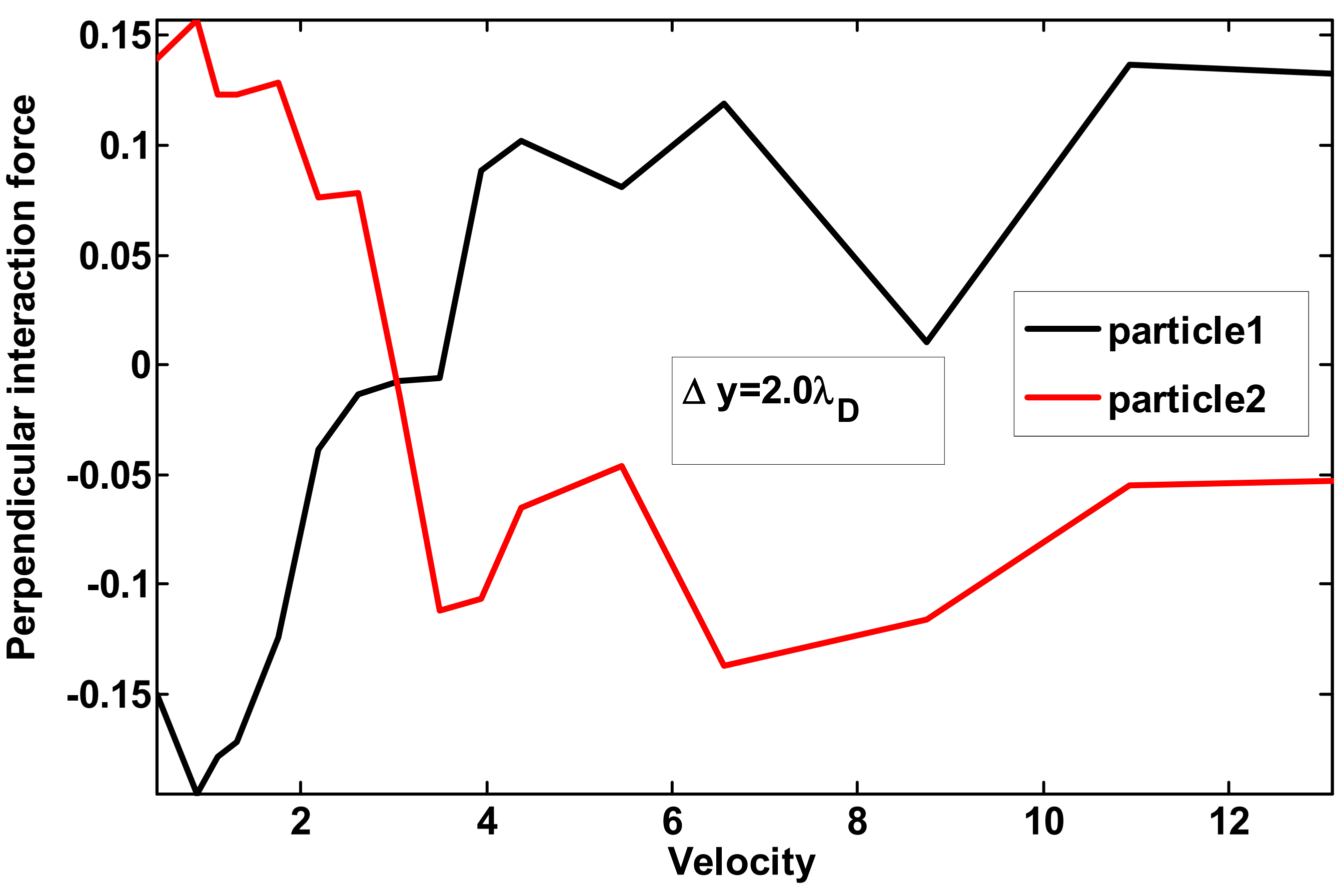
Transversal forces



Transversal forces



Transversal forces



Thanks for

your attention!

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Analytical models of dust layer response

- Hydrodynamic model, gives the RPA dielectric function
- Dielectric response theory using QLCA

Hydrodynamic model

$$\frac{\partial \sigma_d(\mathbf{r},t)}{\partial t} + \nabla_\parallel \cdot [\sigma_d(\mathbf{r},t)\mathbf{u}_d(\mathbf{r},t)] = 0,$$

Correlation effects

$$\frac{\partial \mathbf{u}_d(\mathbf{r},t)}{\partial t} + \mathbf{u}_d(\mathbf{r},t) \cdot \nabla_\parallel \mathbf{u}_d(\mathbf{r},t) = \frac{eZ_d}{m_d}\nabla_\parallel \Phi(\mathbf{R},t)\Big|_{z=0} + \frac{\mathbf{F}_{int}}{m_d} + \frac{eZ_d}{m_d c}[\mathbf{u}_d(\mathbf{r},t)\mathbf{B}_0] + \frac{\mathbf{F}_{ext}}{m_d} - \gamma \mathbf{u}_d(\mathbf{r},t),$$

$$\nabla^2 \Phi(\mathbf{R},t) = -4\pi e\,[n_i(\mathbf{R},t) - n_e(\mathbf{R},t) - Z_d \sigma_d(\mathbf{r},t)\delta(z)]\,,$$

$$n_e = n_0 \exp(e\Phi/k_B T_e),\ n_i = n_0 \exp(-e\Phi/k_B T_i)$$

Linearization: $\Phi(\mathbf{R},t) = \Phi_0(z) + \Phi_1(\mathbf{R},t),$ etc.

Response of the dust layer

$$\Phi_{\text{ind}}(\mathbf{K},\omega)=\left[\frac{1}{\varepsilon_L(\mathbf{k},\omega)}-1\right]\Phi_{\text{ext}}(\mathbf{K},\omega)$$

$$\Phi_{\text{ext}}(\mathbf{r},z,t)=\frac{Q_t Q_d}{\sqrt{(\mathbf{r}-\mathbf{v}t)^2+(z-h)^2}}e^{\left(-\kappa\sqrt{(\mathbf{r}-\mathbf{v}t)^2+(z-h)^2}\right)}$$

$$\mathbf{k}=\{k_x,k_y\}$$

$$\mathbf{K}=\{k_x,k_y,k_z\}$$

Dielectric function of the system

$$\varepsilon_L(\mathbf{k},\omega) = 1 - \frac{\omega_0^2(\mathbf{k})}{\omega^2 - (\sigma_{d0}/m_d)G(\mathbf{k},\omega)}$$

$$\omega_0^2(\mathbf{k}) = \frac{\omega_{pd}^2 (k\lambda_D)^2}{\sqrt{1+(k\lambda_D)^2}} \qquad \omega_{pd}^2 = \frac{2\pi Q_d^2 \sigma_{d0}}{m_d \lambda_D}$$

$$G(\mathbf{k},\omega) = \begin{cases} 0 & \text{RPA} \\ D_L(\mathbf{k}) & \text{QLCA} \\ \ldots & \end{cases}$$

Formulae

$$\Phi_{\text{ind}}(\mathbf{r},z,t)=\frac{1}{(2\pi)^4}\int d^3\mathbf{K}\,d\omega\,\Phi_{\text{ind}}(\mathbf{K},\omega)e^{i\mathbf{K}\cdot\mathbf{R}-i\omega t}$$

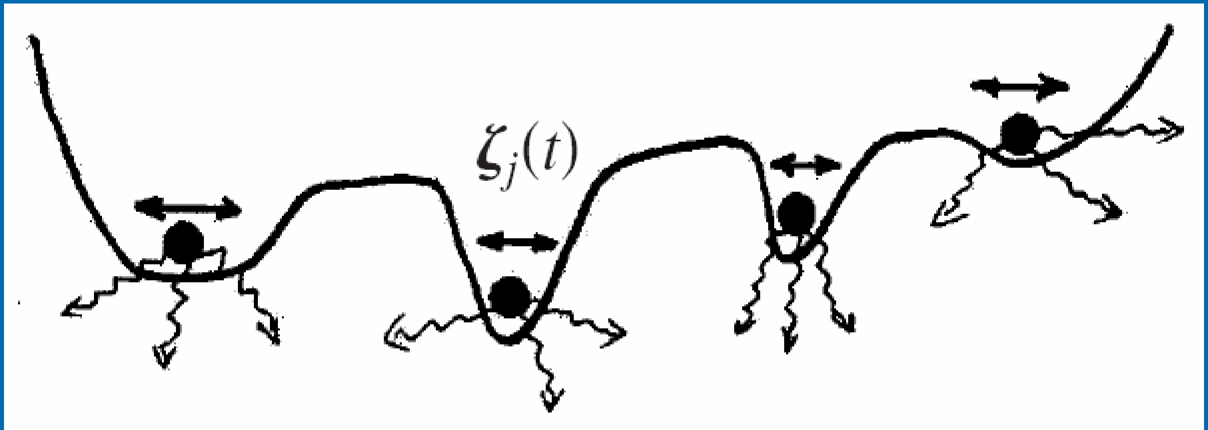
$$F_{st}(v)=Q_t\frac{\partial\Phi_{\text{ind}}(\mathbf{r},z,t)}{\partial x}\bigg|_{z=h,\mathbf{r}=\mathbf{v}t}$$

Stopping force (power)

$$F_{im}(v)=Q_t\frac{\partial\Phi_{\text{ind}}(\mathbf{r},z,t)}{\partial z}\bigg|_{z=h,\mathbf{r}=\mathbf{v}t}$$

Image force

Quasi-Localized Charge Approxim.

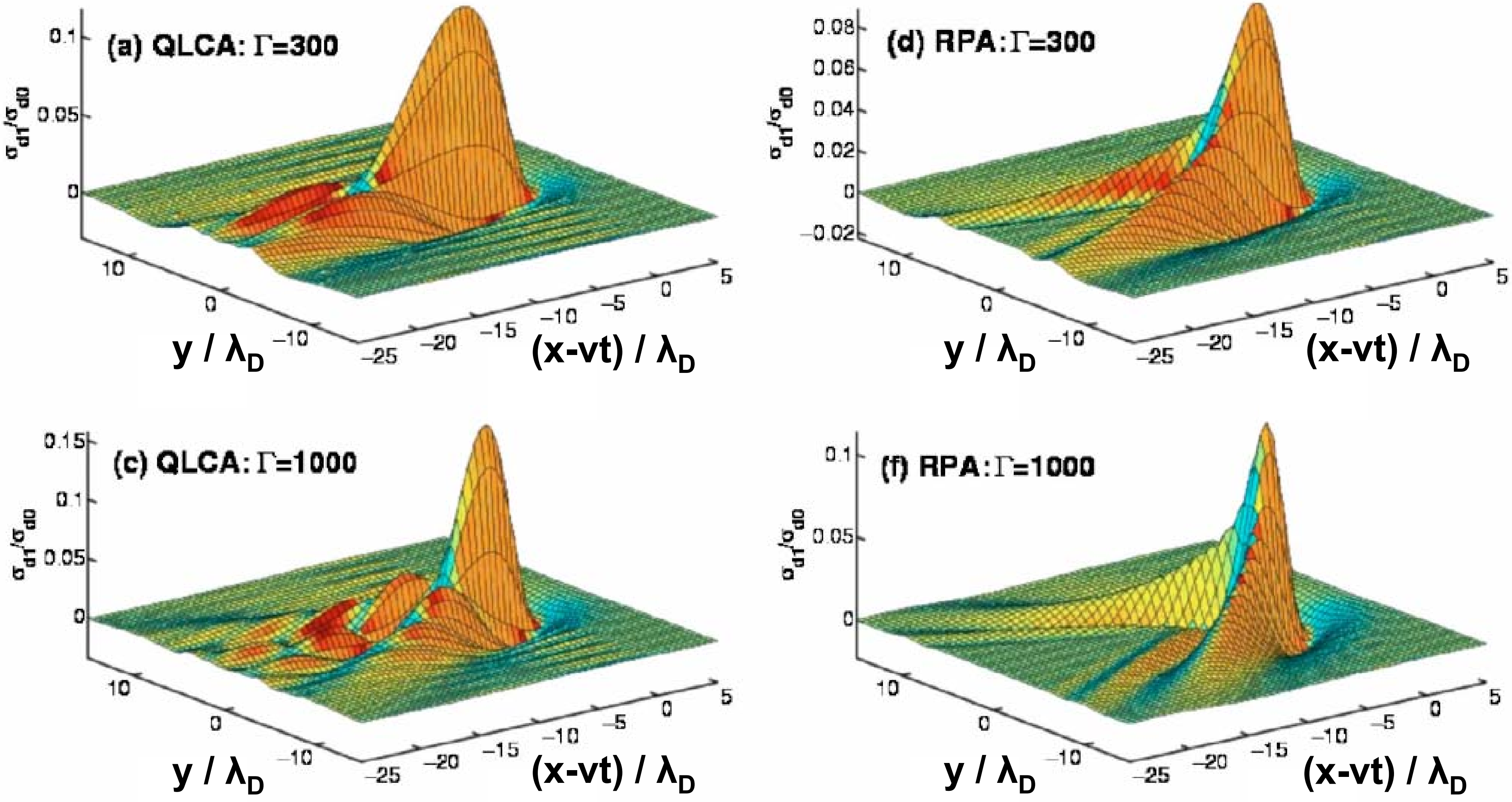


$$\sigma_{d1}(\mathbf{r},t)=\sum_{j=1}^{N_d}\langle\rho(\mathbf{r}_j+\zeta_j(t))-\rho(\mathbf{r}_j)\rangle,\quad \mathbf{u}_{d1}(\mathbf{r},t)=\sum_{j=1}^{N_d}\left\langle\frac{d\zeta_j(t)}{dt}\right\rangle,$$

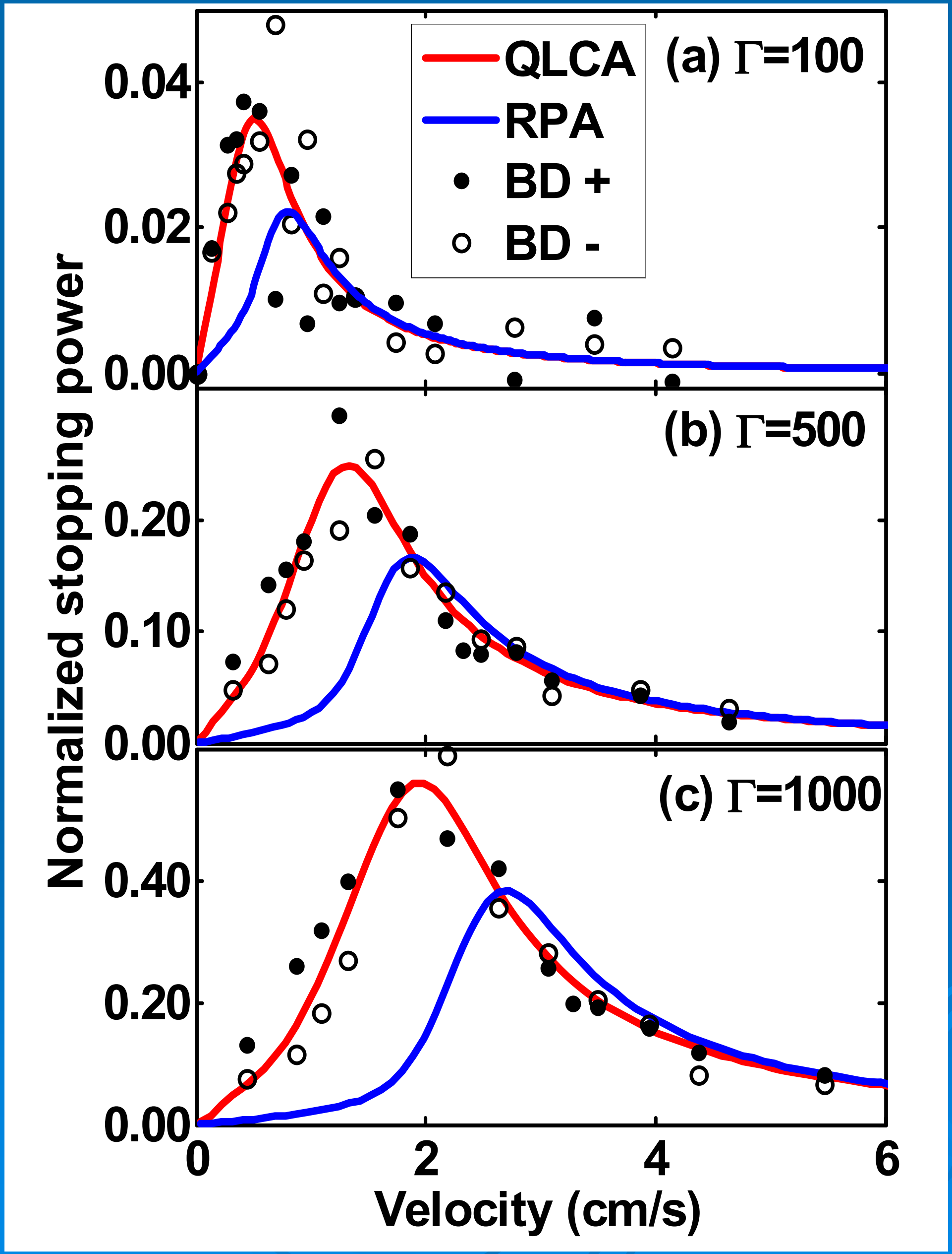
$$-\omega^2\zeta_{\mathbf{k}}(\omega)=-\left[\mathbf{D}(\mathbf{k})+\frac{\sigma_{d0}\phi(k)}{m_d}\mathbf{kk}\right]:\zeta_{\mathbf{k}}(\omega)+i\gamma\omega\zeta_{\mathbf{k}}(\omega)+\frac{\sigma_{d0}}{(m_dN_d)^{1/2}}\mathbf{F}_{ext}(\mathbf{k},\omega),$$

$$D_L(k)=\frac{\omega_{pd}^2\lambda_D^2}{2}\int_0^\infty dr\frac{g(r)-1}{r^2}\exp\left(-\frac{r}{\lambda_D}\right)\left[\left(1+\frac{r}{\lambda_D}+\frac{r^2}{\lambda_D^2}\right)-\left(4+\frac{4r}{\lambda_D}+\frac{2r^2}{\lambda_D^2}\right)J_0(kr)+\left(6+\frac{6r}{\lambda_D}+\frac{2r^2}{\lambda_D^2}\right)\frac{J_1(kr)}{kr}\right],$$

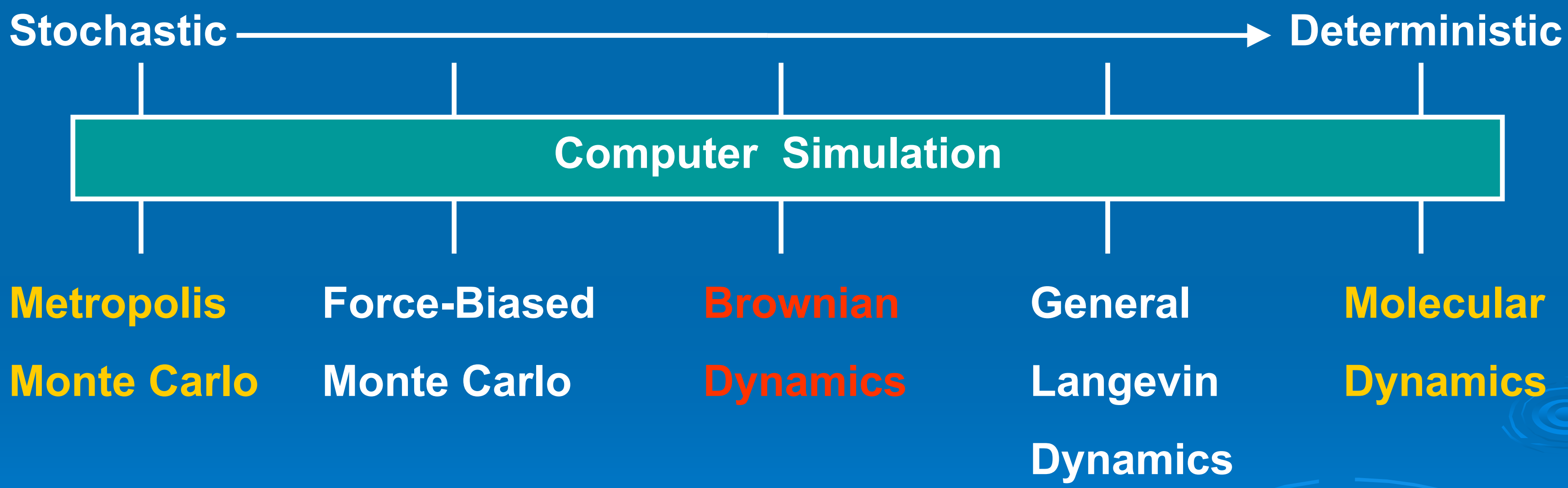
Induced dust density: QLCA vs. RPA



Mean stopping force



Brownian Dynamics simulation



BD simulation

- Based on Langevin equation

$$\frac{d}{dt}\mathbf{v}(t) = -\gamma\mathbf{v}(t) + \frac{1}{m}\mathbf{F}(\mathbf{r}(t)) + \mathbf{A}(t)$$

$$\frac{d}{dt}\mathbf{r}(t) = \mathbf{v}(t)$$

γ

$\mathbf{A}(t)$

Fluctuation

Dissipation

$\rightarrow k_B T$

Algorithms for BD simulation

- Euler-like
  - Ermak, J. Chem. Phys. 62, 4189 (1975)
- Beeman-like:
  - Allen, Mol. Phys., 66, 3039 (1980)
- Verlet-like
  - Van Gunsteren and Berendsen, Mol. Phys. 45, 637 (1982)
- Gear-Like Predictor-Corrector
  - Hou, Miskovic and Wang, in preparation

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Euler-like method

$$\mathbf{v}(t) = \mathbf{v}_0 e^{-\gamma t} + \frac{t}{m}\mathbf{F}_0 \frac{1-e^{-\gamma t}}{\gamma t} + \mathbf{R}_{\mathbf{v}}(t),$$

$$\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}_0 \frac{1-e^{-\gamma t}}{\gamma t} + \frac{t^2\mathbf{F}_0}{m\gamma t}\left[1-\frac{1-e^{-\gamma t}}{\gamma t}\right] + \mathbf{R}_{\mathbf{r}}(t),$$

$$\mathbf{R}_{\mathbf{v}}(t) = \sqrt{\frac{k_B T}{m}(1-e^{-\gamma t})}\mathbf{N}_{\mathbf{v}}(0,1);$$

$$\mathbf{R}_{\mathbf{r}}(t) = \sqrt{\frac{2k_B T}{m}[1-2\frac{1-e^{-\gamma t}}{\gamma t}+\frac{1-e^{-2\gamma t}}{2\gamma t}]}\mathbf{N}_{\mathbf{r}}(0,1)$$

Euler-like method

$\mathbf{F}_0 = \mathbf{F}(0)$ is a constant

Euler-like method: when $\gamma \to 0$ it recovers the Euler method

$$\mathbf{r}(t) = \mathbf{a}_0 + a_1 \mathbf{N}_1(0,1)$$

$$\mathbf{v}(t) = \mathbf{b}_0 + b_1 \mathbf{N}_1(0,1) + b_2 \mathbf{N}_2(0,1),$$

$$\mathbf{a}_0 = \mathrm{mean}\{\mathbf{r}\}; \qquad \mathbf{a}_1^2 = \mathrm{var}\{\mathbf{r}\};$$

$$\mathbf{b}_0 = \mathrm{mean}\{\mathbf{v}\}; \quad \mathbf{b}_1^2 = \frac{\mathrm{cov}\{\mathbf{v},\mathbf{r}\}}{\sqrt{\mathrm{var}\{\mathbf{r}\}}}; \qquad \mathbf{b}_2^2 = \mathrm{var}\{\mathbf{v}\} - \mathbf{a}_1^2$$

Beeman-like method

$$\mathbf{F} \approx \mathbf{F}(0) + \mathbf{F}'(0)t \qquad \mathbf{F}'(0) \approx \left[\mathbf{F}(0) - \mathbf{F}(-t)\right]/t$$

$$\mathbf{r}(t) = \mathbf{a}_0 + a_1\mathbf{N}_{\mathbf{v}}(0,1)$$

$$\mathbf{v}(t) = \mathbf{b}_0 + b_1\mathbf{N}_{\mathbf{v}}(0,1) + b_2\mathbf{N}_{\mathbf{r}}(0,1),$$

$$\mathbf{a}_0 = \mathrm{mean}\{\mathbf{r}\} = \mathbf{r}_0 + c_a t\mathbf{v}_0 + c_b t^2\frac{\mathbf{F}(0)}{m} + c_c t^2\frac{\mathbf{F}(-t)}{m},$$

$$\mathbf{b}_0 = \mathrm{mean}\{\mathbf{v}\} = c_d\mathbf{v}_0 + c_e t\frac{\mathbf{F}(t)}{m} + c_f t\frac{\mathbf{F}(0)}{m} + c_g t\frac{\mathbf{F}(-t)}{m}$$

Beeman-like method

when $\gamma \to 0$ it recovers the Beeman method

$$c_a = c_1, \qquad c_d = c_0, \qquad c_0 = e^{-\gamma t}$$

$$c_b = c_2 + c_3, \qquad c_e = c_2 - c_0 c_3 / c_1, \qquad c_1 = (1 - c_0) / \gamma t$$

$$c_c = -c_3, \qquad c_f = c_1 - c_2 + 2 c_0 c_3 / c_1, \qquad c_2 = (1 - c_1) / \gamma t$$

$$c_g = -c_0 c_3 / c_1, \qquad c_3 = (1/2 - c_2) / \gamma t$$

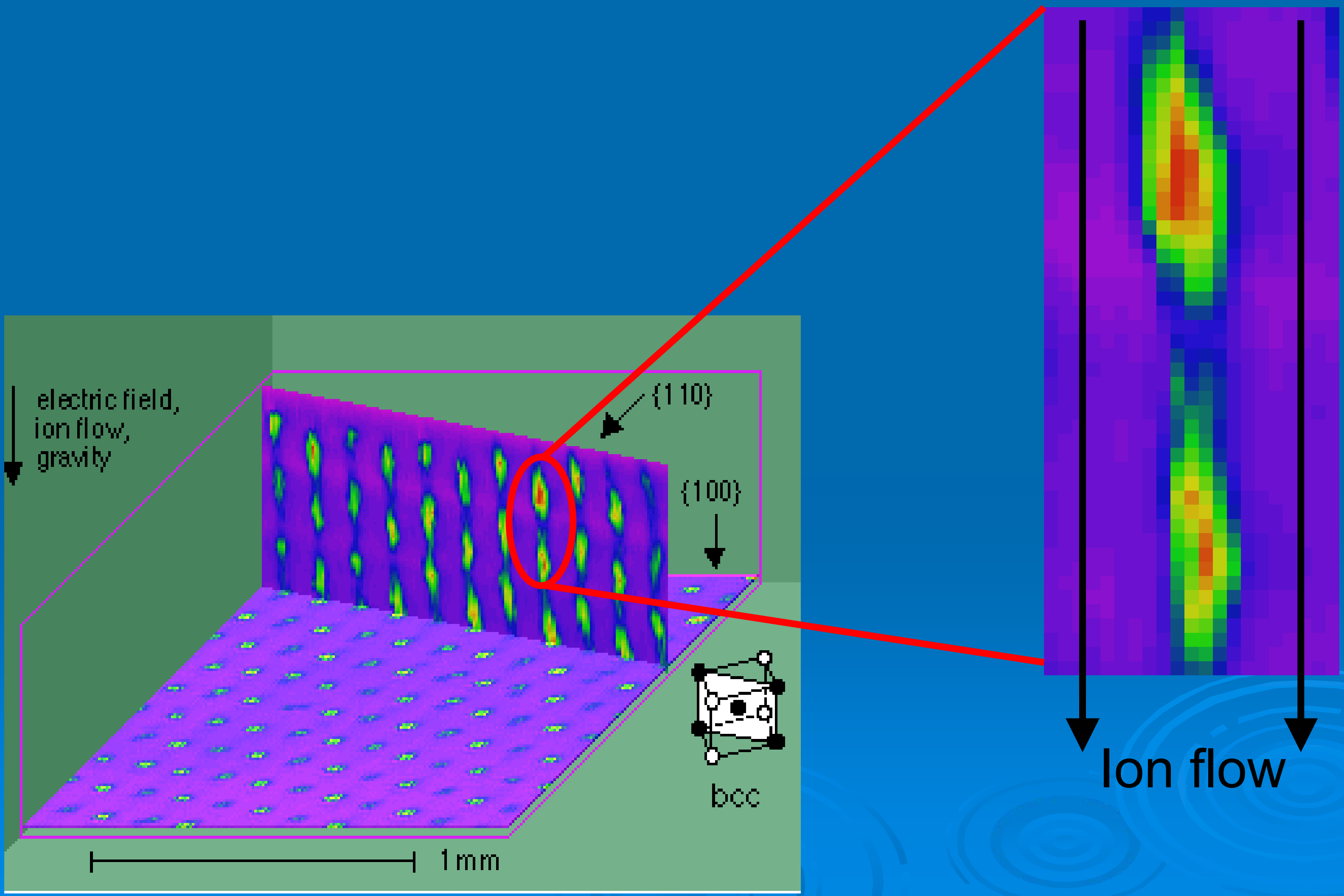
Boundary and initial conditions

- Boundary conditions
  - Periodic boundary with a force cutoff
- Initial conditions
  - Random positions and velocities, or
  - Previous results
- Particle number: N=1000~2000

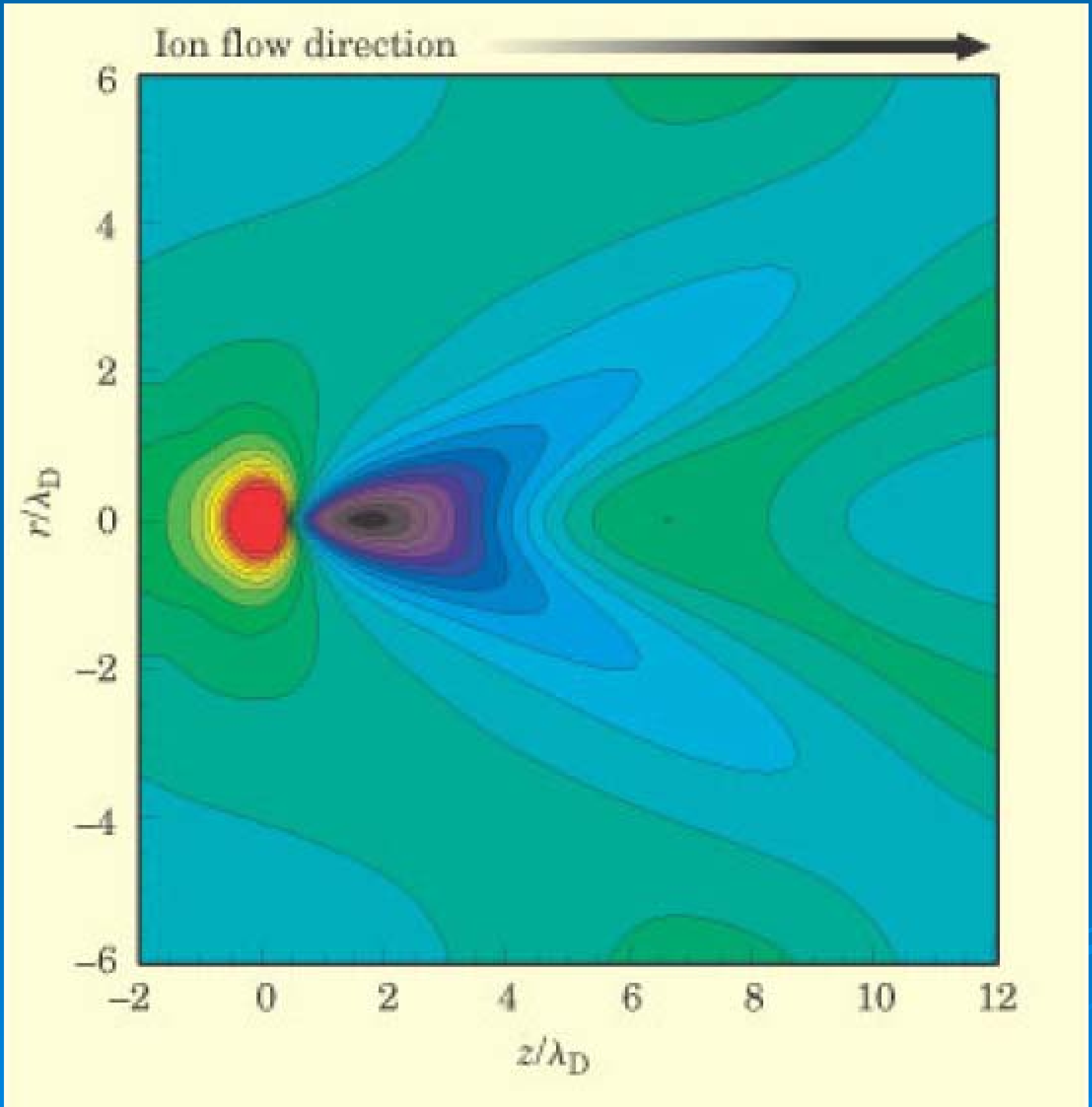
Going beyond Yukawa inter-dust interaction potential

Dust particles immersed in plasma sheath with ion flow and non-homogeneous distribution of electron & ion density, and electric field

Ion wake: vertical alignment of dust particles in plasma sheath

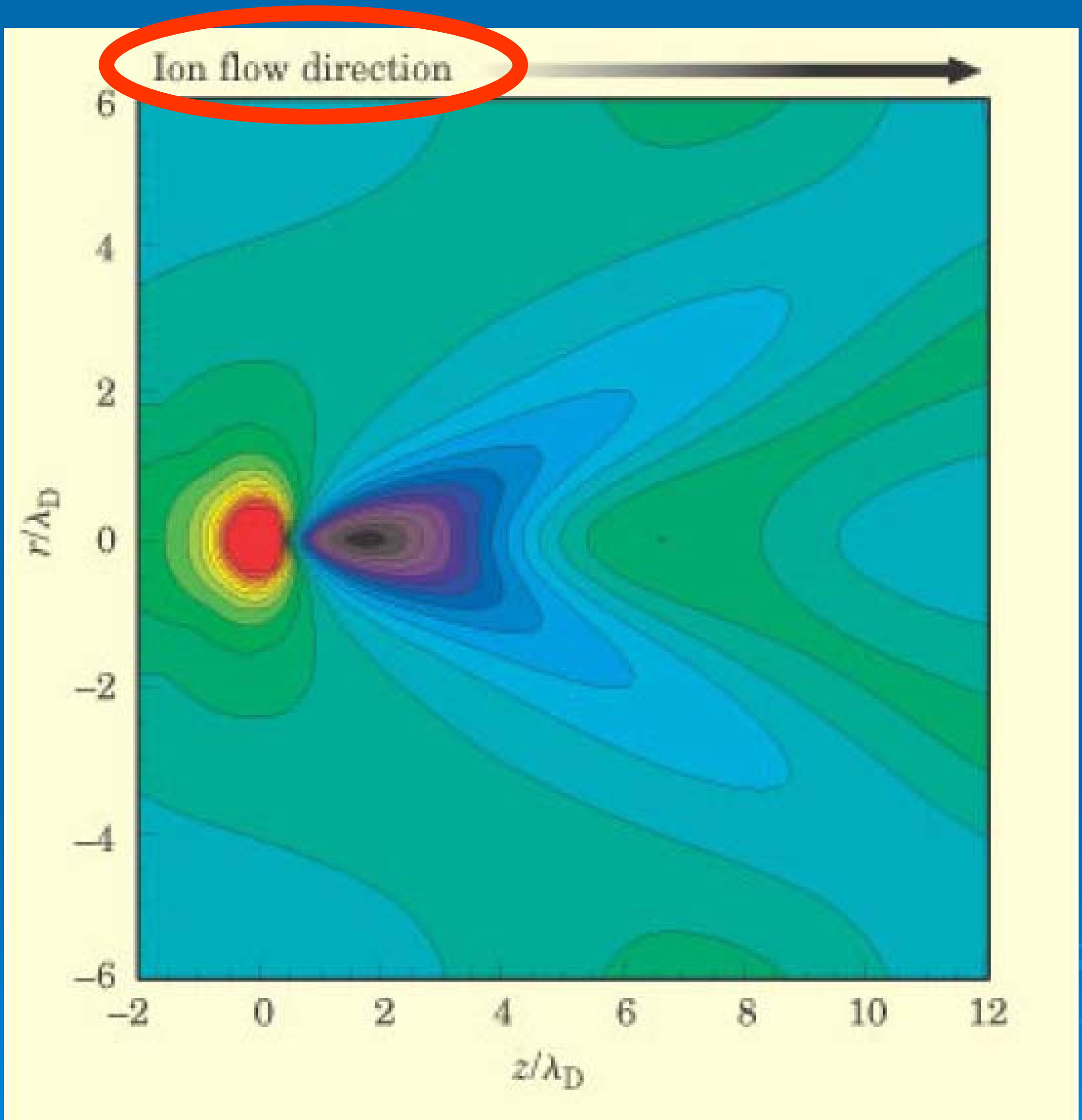


Ion wake and inter-dust interactions



Equipotential curves from PIC simulation by Lampe *et al.*

Ion wake and inter-dust interactions



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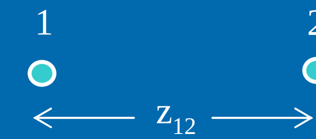
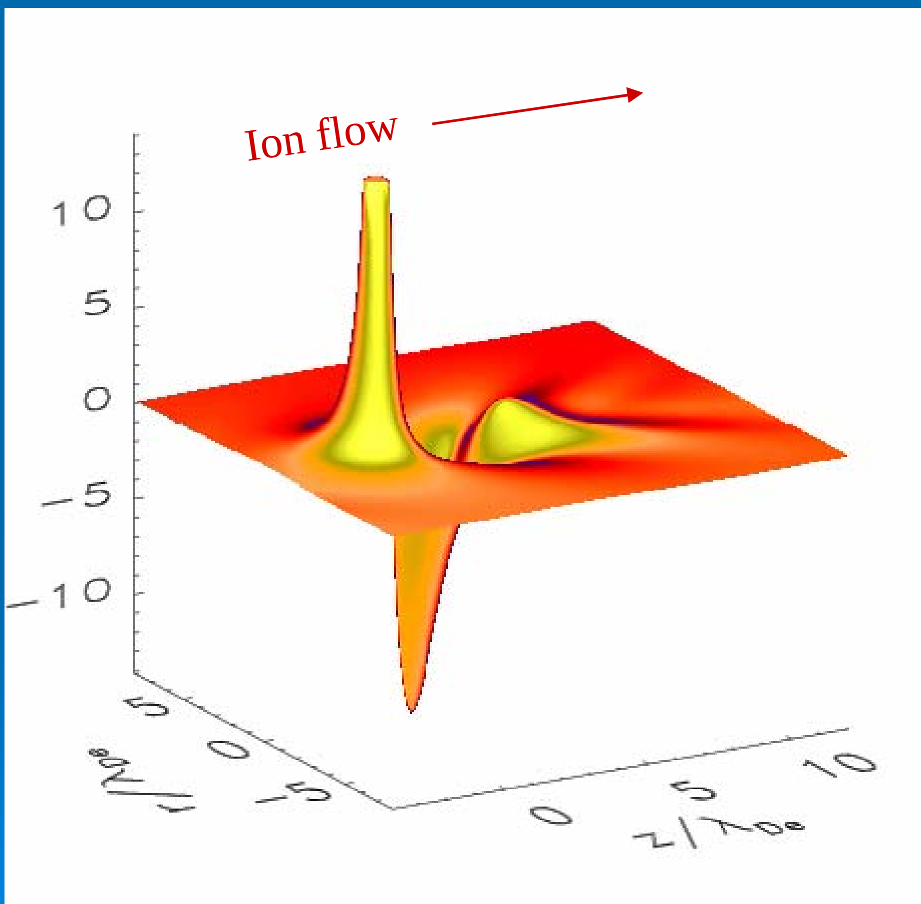
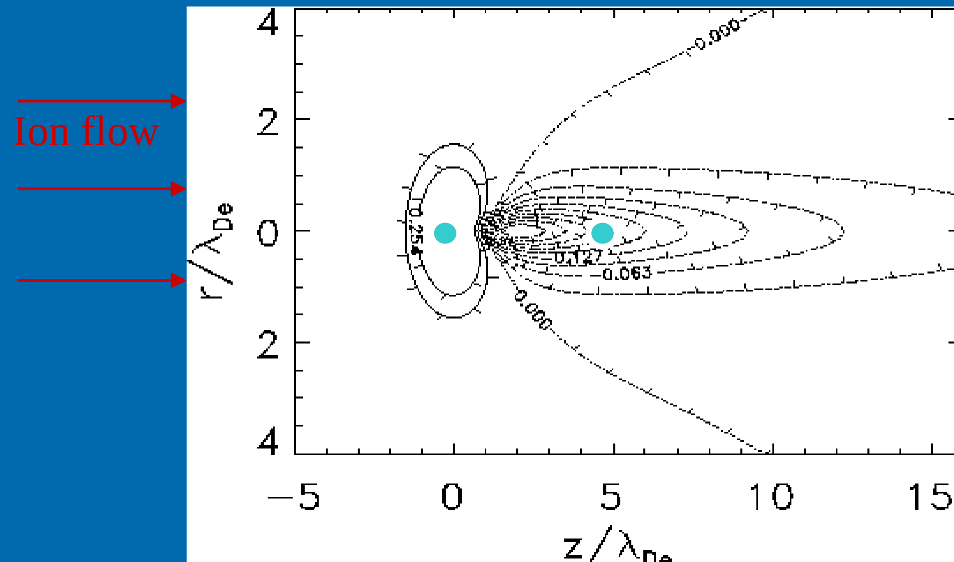
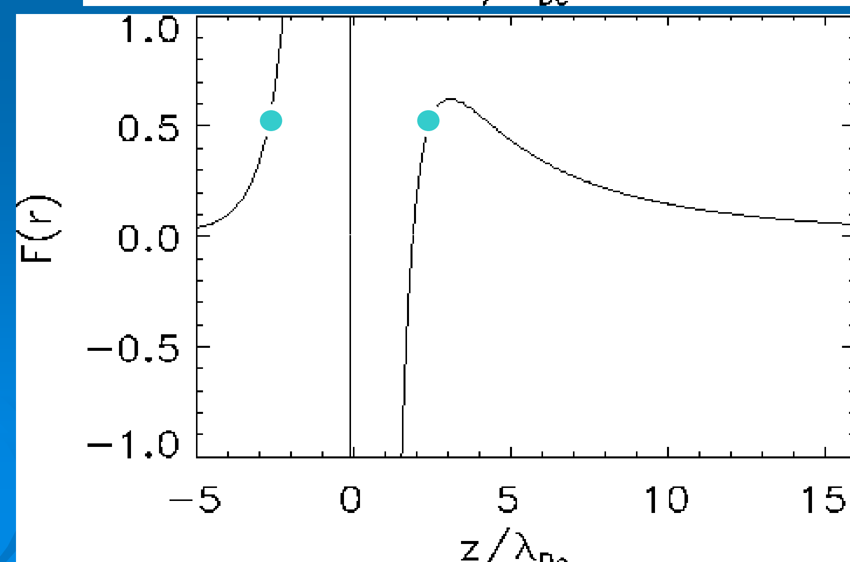






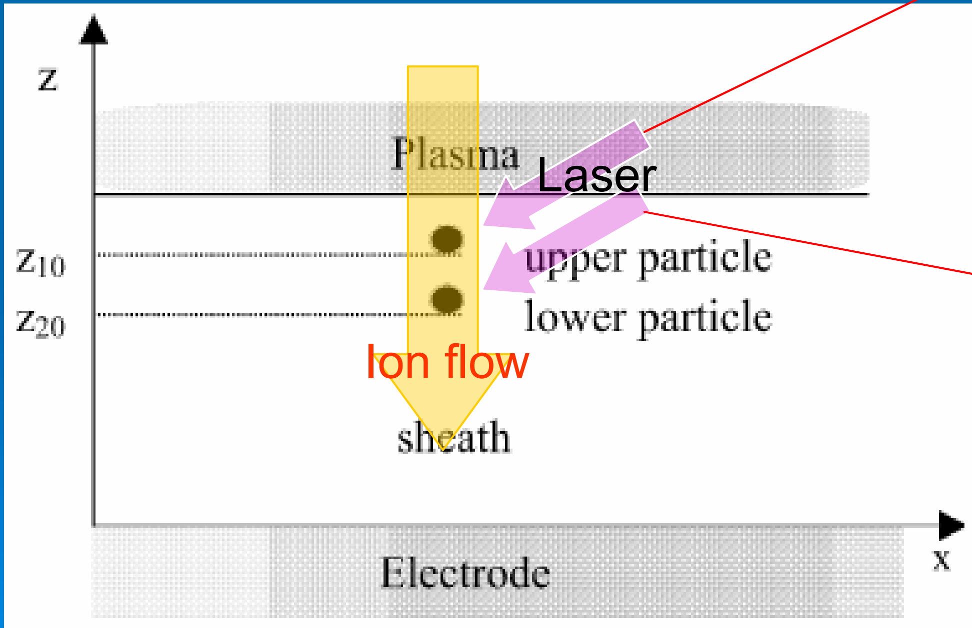


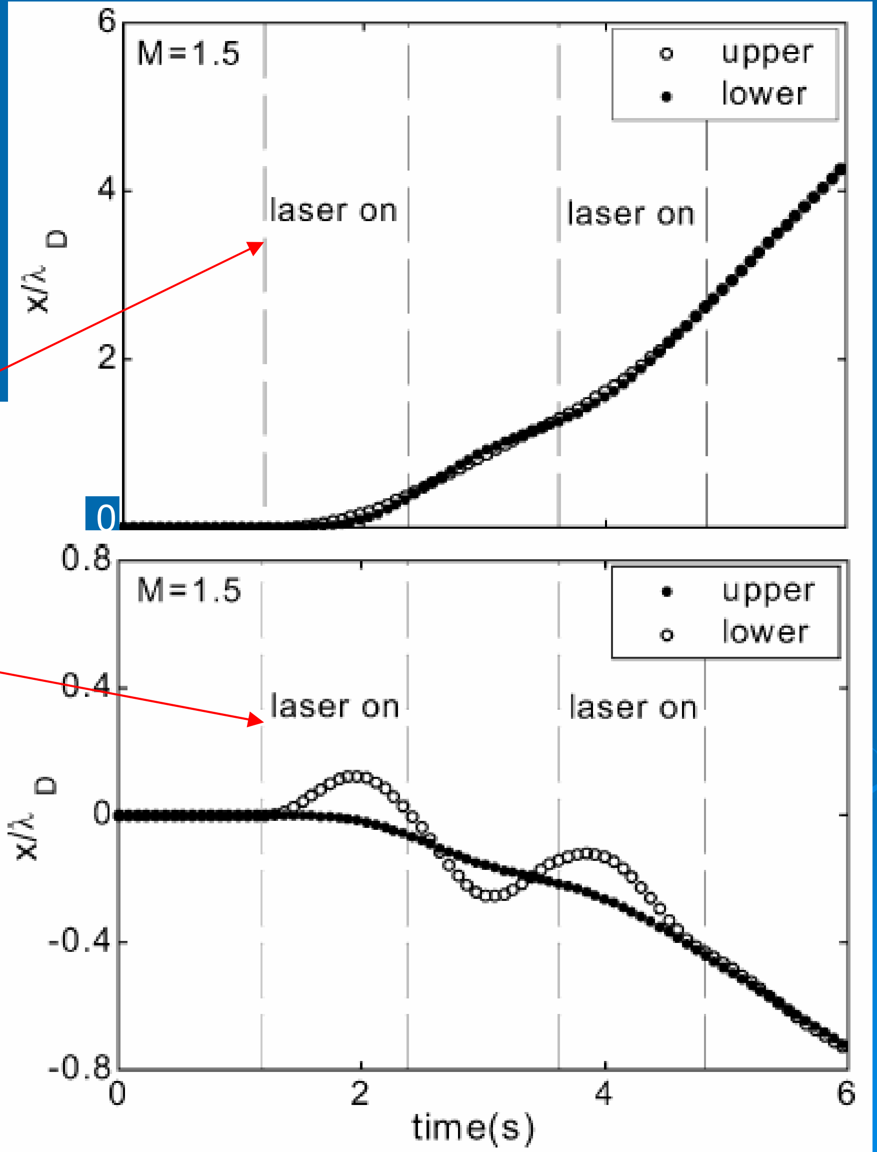
Wake riding effect for two particles in sheath

Find horizontal positions of particles from

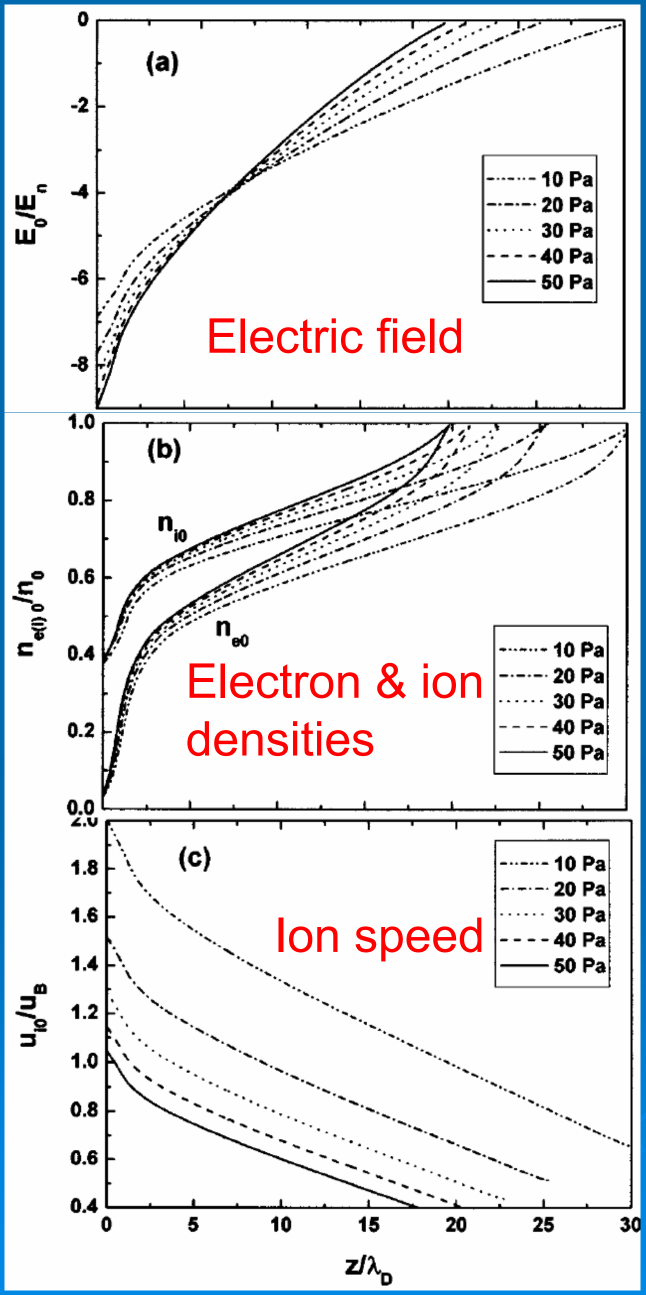
$$m_i \frac{d^2 x_i}{dt^2} = F_x(x_{ij}, z_{ij}) + F_n(u_{ix}) + F_{opt}$$

for $z_{ij} = const.$, F_x exhibits asymmetry of wake between upper and lower particles





Effects of sheath on ion wake



Hydrodynamic model for ions

$$\nabla \cdot [n_i(\mathbf{r})\mathbf{u}_i(\mathbf{r})] = 0$$

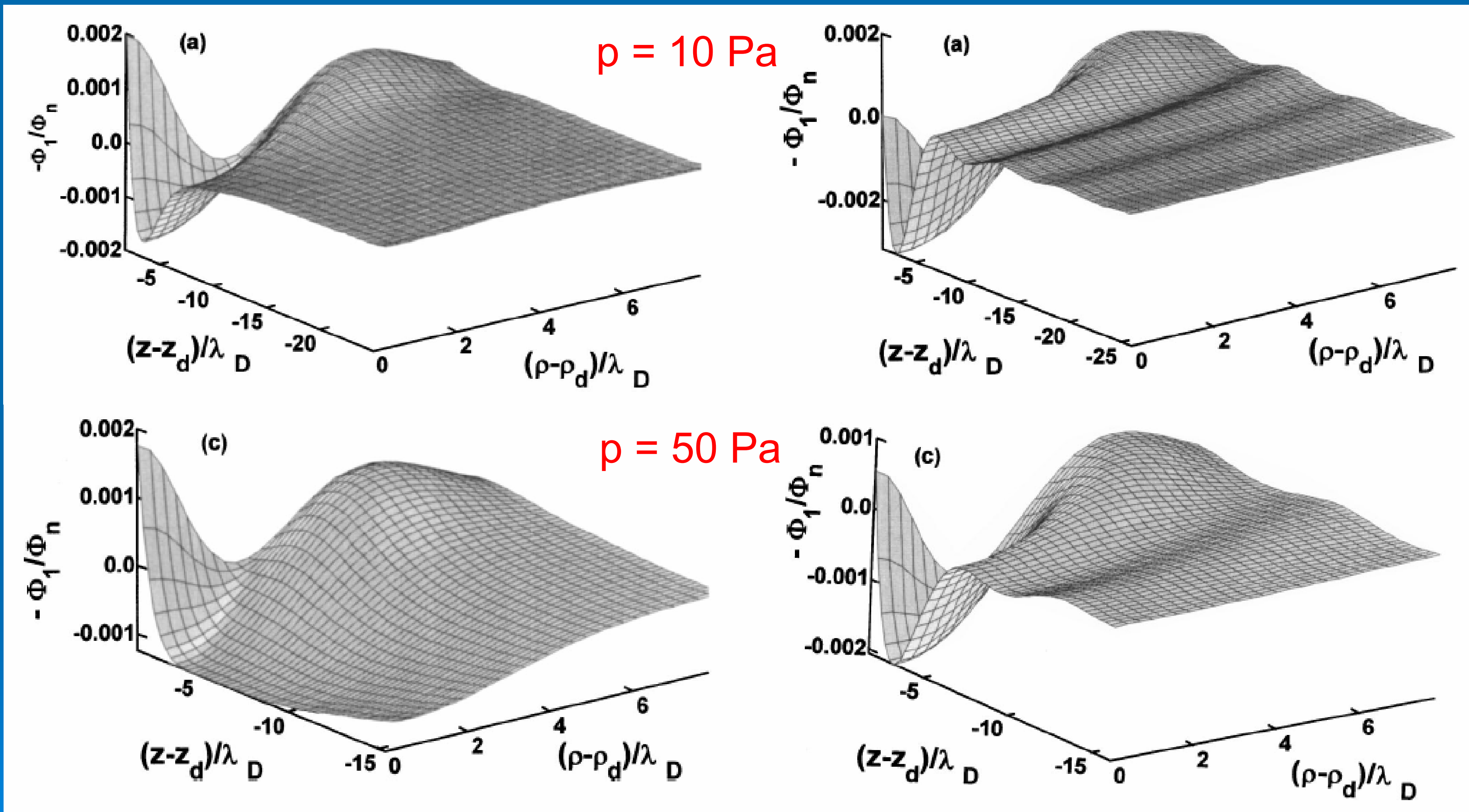
$$\mathbf{u}_i(\mathbf{r}) \cdot \nabla \mathbf{u}_i(\mathbf{r}) = -\frac{Z_i e}{m_i} \nabla \Phi(\mathbf{r}) - \nu \mathbf{u}_i(\mathbf{r})$$

$$\nabla^2 \Phi(\mathbf{r}) = -4\pi[e n_i(\mathbf{r}) - e n_e(\mathbf{r}) - Q_d \delta(\mathbf{r} - \mathbf{r}_d)]$$

Linearize about sheath values for:

$$n_{i0}(z),\ u_{i0}(z),\ n_{e0}(z),\ \mathbf{E}_0(z)$$

Effects of sheath on ion wake



Inhomogeneous sheath

Homogeneous plasma

Thanks for

your attention!