

# Dynamics of fast molecular ions in solids and plasmas

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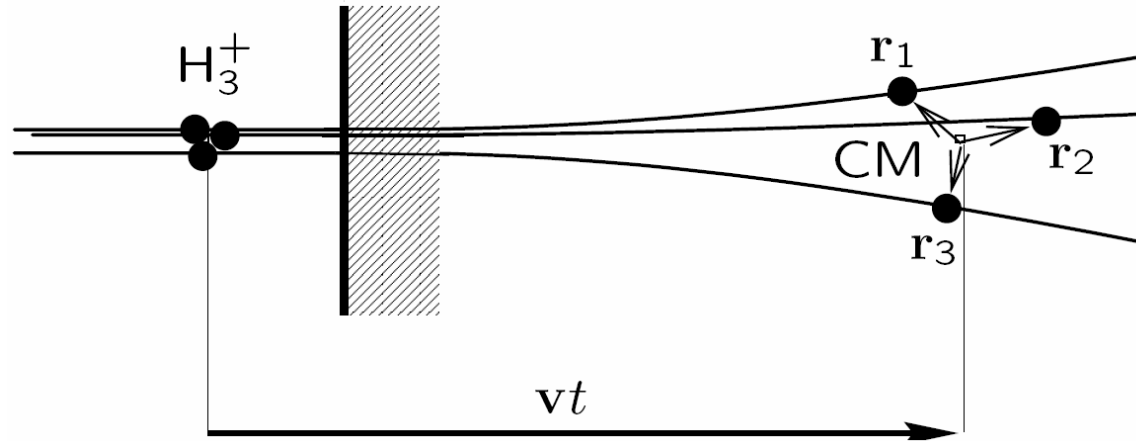
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*Support:* NSERC, PREA, NNSFC, NEMC

# Outline

- ❑ **Introduction**
- ❑ **Charge states**
- ❑ **Coulomb explosions**
  - solids
  - plasmas
  - effects of laser
- ❑ **Grazing scattering from surfaces**
- ❑ **More exotic cases**
  - Dust particles in complex plasmas
  - Channeling through carbon nanotubes
- ❑ **Outlook**

# Cluster penetration through target



- Fast clusters:  $v > v_{Bohr}$  or  $v > v_{Therm}$
- Adiabatic evolution of structure in CM frame:  $\left| \frac{d\mathbf{r}_j}{dt} \right| \ll v$
- Rapid ionization ( $< 1$  fs) followed by charge quasi-equilibrium:  $\frac{d}{dt} \langle Q_j \rangle \approx 0$
- Vicinage effects due to  $|\mathbf{r}_j - \mathbf{r}_l| \propto v / \omega_{res}$  affect:
  - Cluster stopping power
  - Coulomb explosion
  - Ion charge states

# Dielectric theory of cluster stopping power

Review: N.R. Arista, NIMB 164-165 (2000) 108

Decomposition  $S_{cl} = S_{incoh} + S_{coh}$  gives stopping ratio  $\mathcal{R}_S = 1 + \frac{S_{coh}}{S_{incoh}}$

$$S_{incoh} = \sum_{j=1}^n \int \frac{d^3\mathbf{k}}{2\pi^2} |\rho_j(\mathbf{k})|^2 \frac{\mathbf{k} \cdot \mathbf{v}}{k^2 v} \text{Im} \left[ \frac{-1}{\epsilon(k, \mathbf{k} \cdot \mathbf{v})} \right]$$

$$S_{coh} = \sum_{j=1}^n \sum_{l \neq j=1}^n \int \frac{d^3\mathbf{k}}{2\pi^2} \rho_j(\mathbf{k}) \rho_l^*(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}_{jl}} \frac{\mathbf{k} \cdot \mathbf{v}}{k^2 v} \text{Im} \left[ \frac{-1}{\epsilon(k, \mathbf{k} \cdot \mathbf{v})} \right]$$

Charge densities (form factors) of the  $j$ -th and  $l$ -th ions; interference factor

Because ion sizes are  $\ll |\mathbf{r}_{jl}|$ , one can use point-charge approximation in coherent stopping,  $\rho_j(\mathbf{k}) \approx Q_j$  and  $\rho_l(\mathbf{k}) \approx Q_l$ , but **not** in incoherent stopping

# Target polarization effects on Coulomb explosion

Newton's equations in CM:  $m_l \frac{d^2 \mathbf{r}_l}{d\tau^2} = \mathbf{F}_l^s + \sum_{l \neq j=1}^n \mathbf{F}_{jl}$

Stopping force on the  $l$ -th ion

Force on the  $l$ -th ion from the  $j$ -th ion:  $\mathbf{F}_{jl} = -\frac{\partial V_{jl}(\mathbf{r}_{jl})}{\partial \mathbf{r}_l}$

Point-charge approximation in the inter-ion potential:  $\rho_j(\mathbf{k}) \approx Q_j$  and  $\rho_l(\mathbf{k}) \approx Q_l$

$$V_{jl}(\mathbf{r}_{jl}) = \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{4\pi}{k^2} \frac{\rho_j(\mathbf{k}) \rho_l^*(\mathbf{k})}{\epsilon(k, \mathbf{k} \cdot \mathbf{v})} e^{i\mathbf{k} \cdot (\mathbf{r}_l - \mathbf{r}_j)}$$

# Charge on isolated “heavy” ion: Brandt-Kitagawa model

**solids:** W. Brandt and M. Kitagawa, PRB 25 (1982) 5631

**plasmas:** J. D’Avanzo *et al.*, Phys. Plasmas 3 (1996) 3885

$$E_{iso} = c_k \int d^3\mathbf{r} \rho^{5/3}(\mathbf{r}) - \int d^3\mathbf{r} \rho(\mathbf{r}) \frac{Z}{r} + \frac{\lambda}{2} \iint d^3\mathbf{r} d^3\mathbf{r}' \frac{\rho(\mathbf{r})\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

Energy of  $N$  bound electrons in LAB frame:  $E_L = N \frac{v_r^2}{2} + E_{iso}(N)$

Stripping criterion:  $\frac{dE_L}{dN} \equiv \frac{v_r^2}{2} + E'_{iso}(N_0) = 0$

Charge on isolated ion in dynamic quasi-equilibrium:

$$Q_0(v) \equiv Z - N_0$$

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# Vicinage effect on ion charges in cluster with n ions

## Experiment:

- A. Brunelle, S. Della-Negra, J. Depauw, D. Jacquet, Y. Le Beyec, and M. Pautrat, *Phys. Rev. A*, **59** (1999) 4456.
- A. Chiba, Y. Saitoh, and S. Tajima, *Nucl. Instrum. Methods Phys. Res. B*, **232** (2005) 32.

## Theory:

- 1] Z. L. Mišković, S. G. Davison, F. O. Goodman, W.-K. Liu, and Y.-N. Wang, *Phys. Rev. A*, **61** (2000) 62901.
- 2] J.W. Hartman, T.A. Tombrello, S. Bouneau, S. Della Negra, D. Jacquet, Y. Le Beyec, and M. Pautrat, *Phys. Rev. A*, **62** (2000) 43202.
- 3] S. Heredia-Avalos, R. Garcia-Molina, and N.R. Arista, *Europhys. Lett.*, **54** (2001) 729.
- 4] E.Nardi, Z. Zinamon, T.A. Tombrello, and N.M Tanushev, *Phys. Rev. A*, **66** (2002) 13201.
- 5] T. Kaneko, *Phys. Rev. A*, **66** (2002) 52901.



# Vicinage effect on ion charges in cluster with n ions

Z.L. Miskovic *et al.*, PRA 63 (2001) 22901

Total electronic energy in cluster of n ions in LAB frame:

$$\mathcal{E}_{\mathcal{L}} = \sum_{j=1}^n N_j \frac{v_r^2}{2} + \sum_{j=1}^n E_{iso}(N_j) + \frac{1}{2} \sum_{j=1}^n \sum_{l \neq j}^n (Z_j - N_j)(Z_l - N_l) U(\mathbf{r}_{jl})$$

Point-charge approximation in the inter-ion potential:  $\rho_j(\mathbf{k}) \approx Q_j = Z_j - N_j$

$$U(\mathbf{r}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{4\pi}{k^2} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\epsilon(k, \mathbf{k} \cdot \mathbf{v})}$$

(We use both Yukawa potential with  $v/\omega_{res}$ , and potential exhibiting wake)

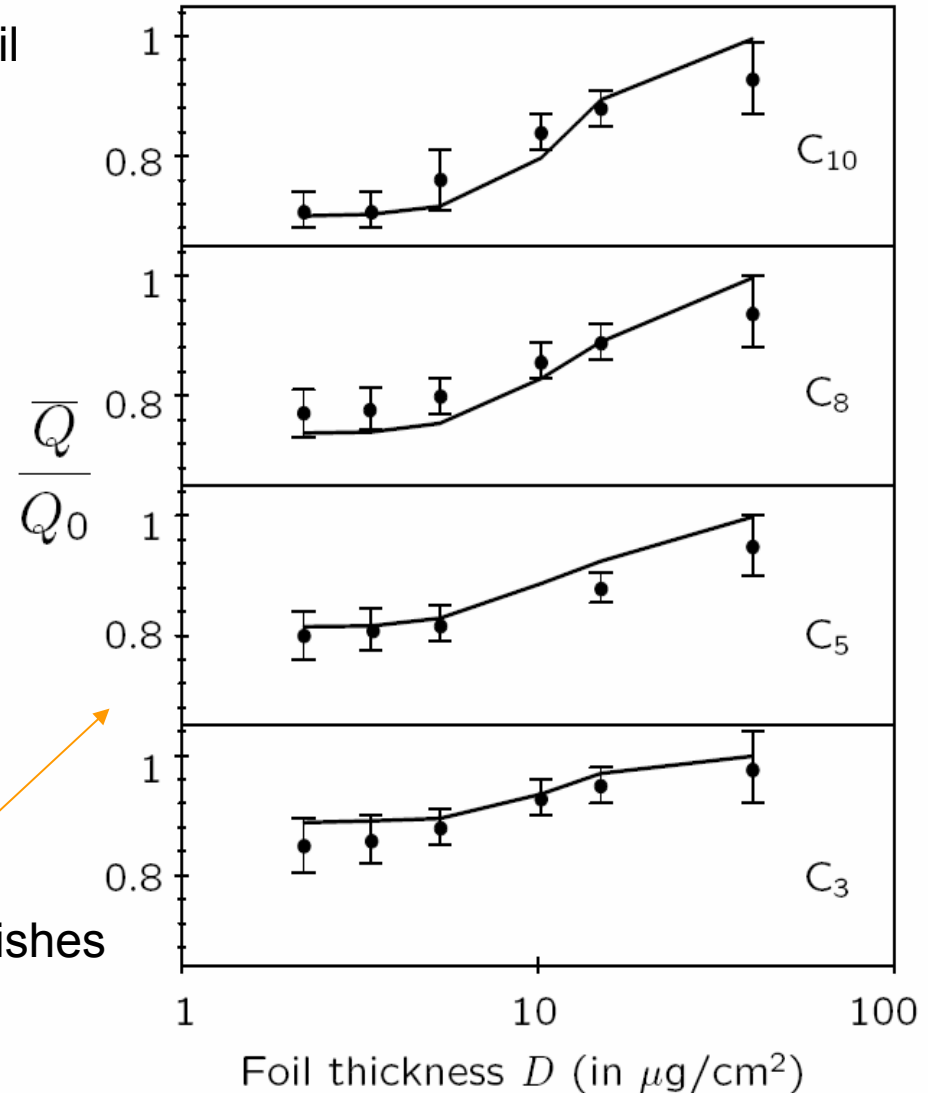
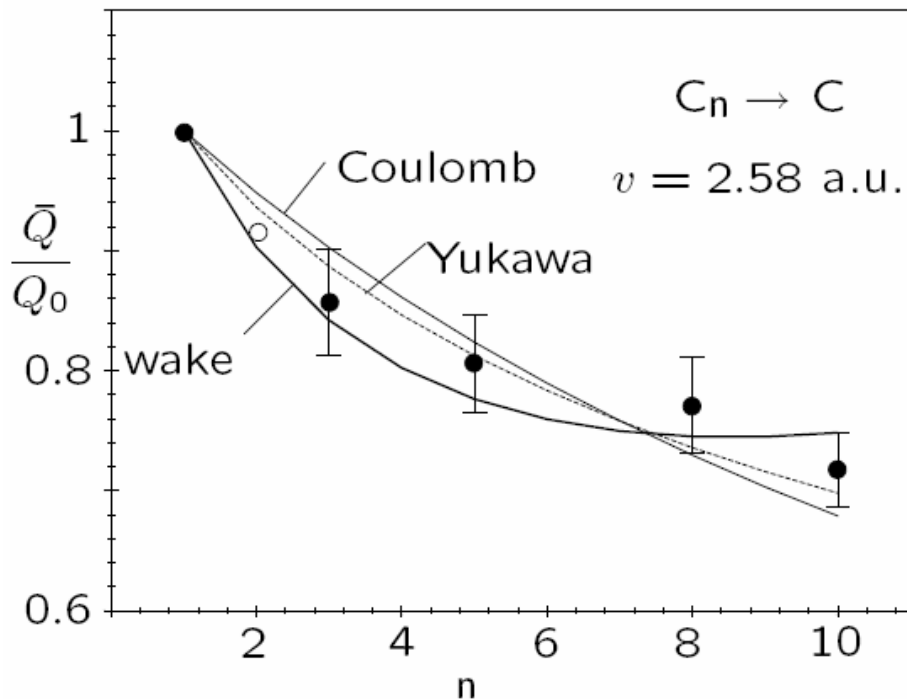
Stripping criterion  $\frac{\partial \mathcal{E}_{\mathcal{L}}}{\partial N_j} = 0$  gives n equations for  $N_1, N_2, \dots, N_n$ :

$$\frac{v_r^2}{2} + E'_{iso}(N_j) - \sum_{j \neq l=1}^n (Z_l - N_l) U(\mathbf{r}_{jl}) = 0$$

# Vicinage effect on ion charges for $C_n^+ \rightarrow$ carbon foil

A. Brunelle *et al.*, PRA 59 (1999) 4456; & A. Chiba *et al.*, NIMB 232 (2005) 32

Charge reduction vs cluster size in thin foil

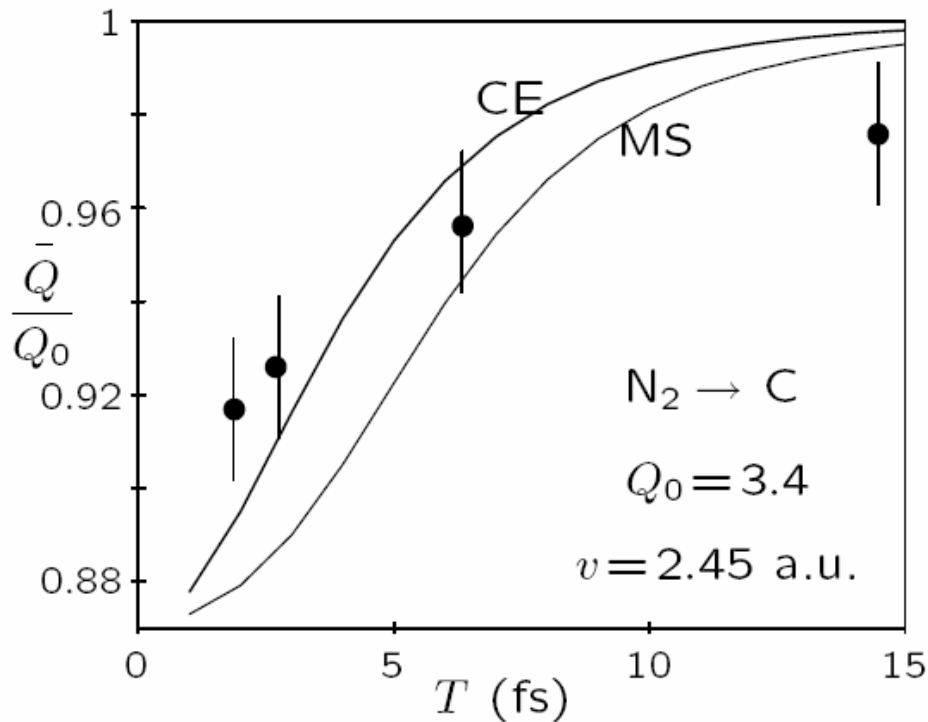


Vicinage effect on ion charges diminishes with increasing foil thickness due to **multiple scattering** on target atoms

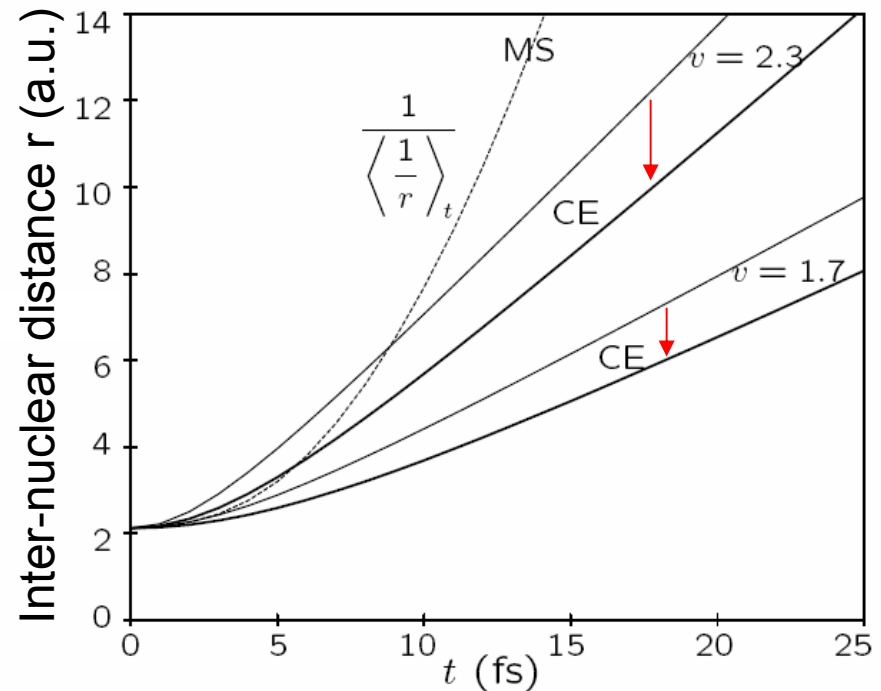
# Vicinage effect on ion charges for $N_2^+ \rightarrow$ carbon foil

D. Maor *et al.*, PRA 32 (1985) 105

Charge reduction vs dwell time due to Coulomb explosion (CE) or multiple scattering (MS)



Charge reduction slows down Coulomb explosion (CE), but **not** multiple scattering (MS)

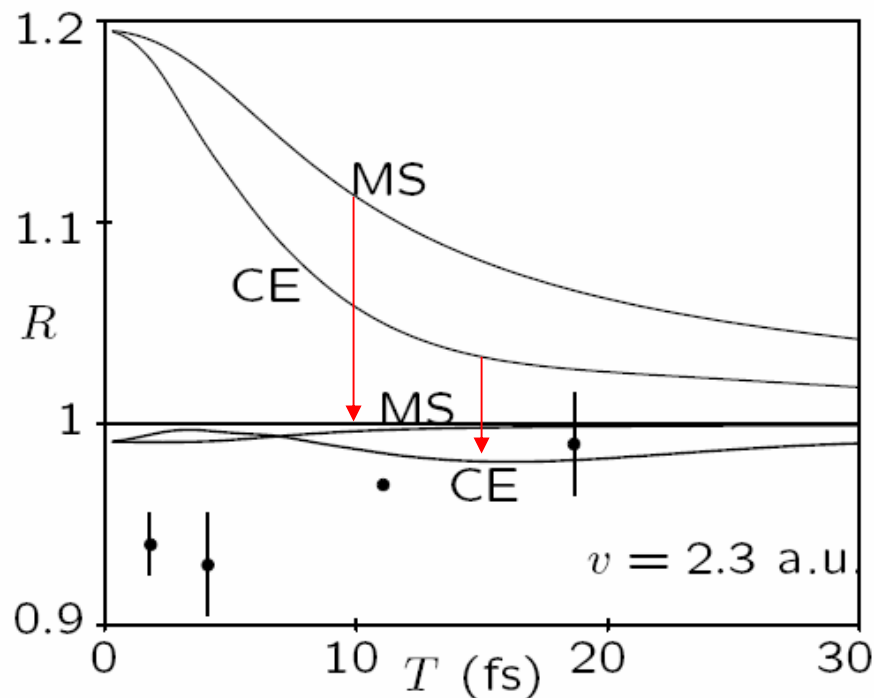
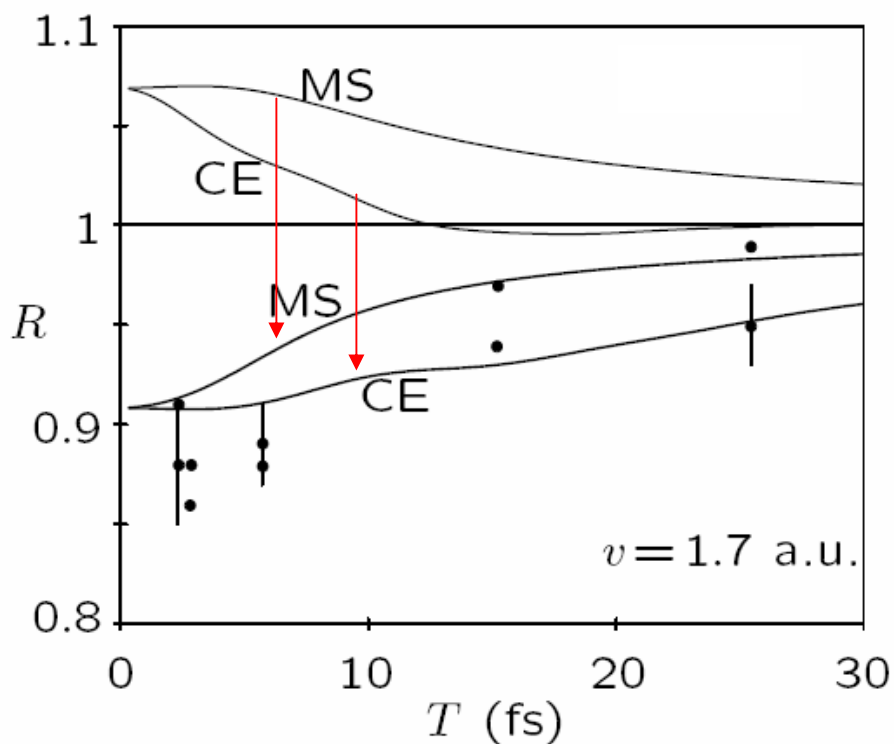


(Charge states and Coulomb explosions evaluated with Yukawa-type potential)

# Vicinage effect on ion charges affects energy losses for fast $N_2^+$ in carbon foil

M.F. Steuer *et al.*, IEEE Trans. Nucl. Sci., NS-30 (1983) 1069

**Conclusion:** charge reduction can change stopping power ratio from  $R > 1$  to  $R < 1$



Alternative explanation: R. Garcia-Molina & S. Heredia-Avalos, PRA 63 (2001) 44901

# Ion charge distribution in **large** homo-nuclear clusters made of heavy ions, passing through very thin foils

Total electronic energy in cluster of  $n$  ions in LAB frame:

$$\langle \mathcal{E}_{\mathcal{L}} \rangle [q(\mathbf{r})] = \frac{v_r^2}{2} \int d^3\mathbf{r} (Z - q(\mathbf{r})) f_1(\mathbf{r}) + \int d^3\mathbf{r} E_{iso}[q(\mathbf{r})] f_1(\mathbf{r}) \\ + \frac{1}{2} \iint d^3\mathbf{r} d^3\mathbf{r}' q(\mathbf{r}) q(\mathbf{r}') U(\mathbf{r} - \mathbf{r}') f_2(\mathbf{r}, \mathbf{r}')$$

Statistical description of cluster structure:  $f_2(\mathbf{r}, \mathbf{r}') = f_1(\mathbf{r}) f_1(\mathbf{r}') g_2(|\mathbf{r} - \mathbf{r}'|)$

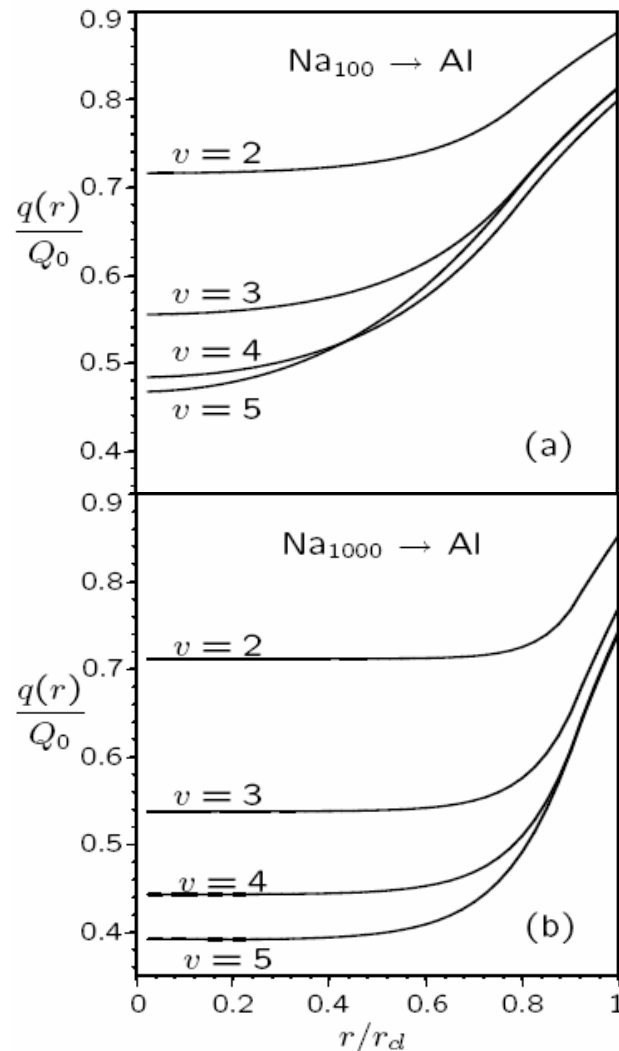
Stripping criterion  $\frac{\delta \langle \mathcal{E}_{\mathcal{L}} \rangle}{\delta q(\mathbf{r})} = 0$  gives integral equation for charge distribution  $q(\mathbf{r})$ :

$$-\frac{v_r^2}{2} + E'_{iso}(q(\mathbf{r})) \\ + (n - 1) \int d^3\mathbf{r}' q(\mathbf{r}') f_1(\mathbf{r}') g_2(|\mathbf{r} - \mathbf{r}'|) U(\mathbf{r} - \mathbf{r}') = 0$$

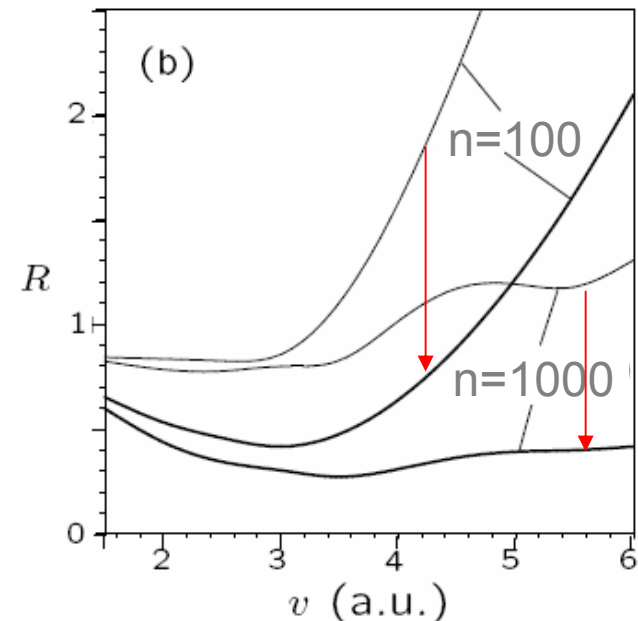
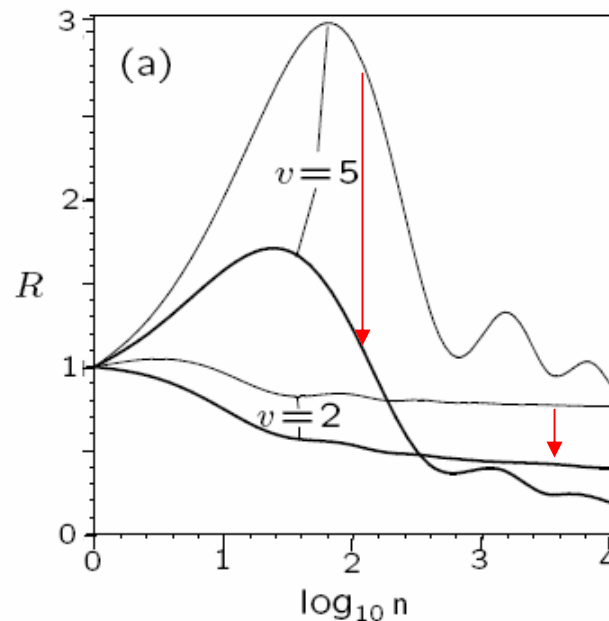
# Ion charges and stopping ratio for $\text{Na}_n^+ \rightarrow \text{Al}$

Z.L. Miskovic *et al.*, PRA 67 (2003) 22903

## Charge distribution through cluster



**Conclusion:** non-homogeneous charge distribution  $q(r)$  can reduce stopping power ratio  $R$  compared to the case of homogeneous distribution where  $q(r) = Q_0 = \text{const.}$  is assumed

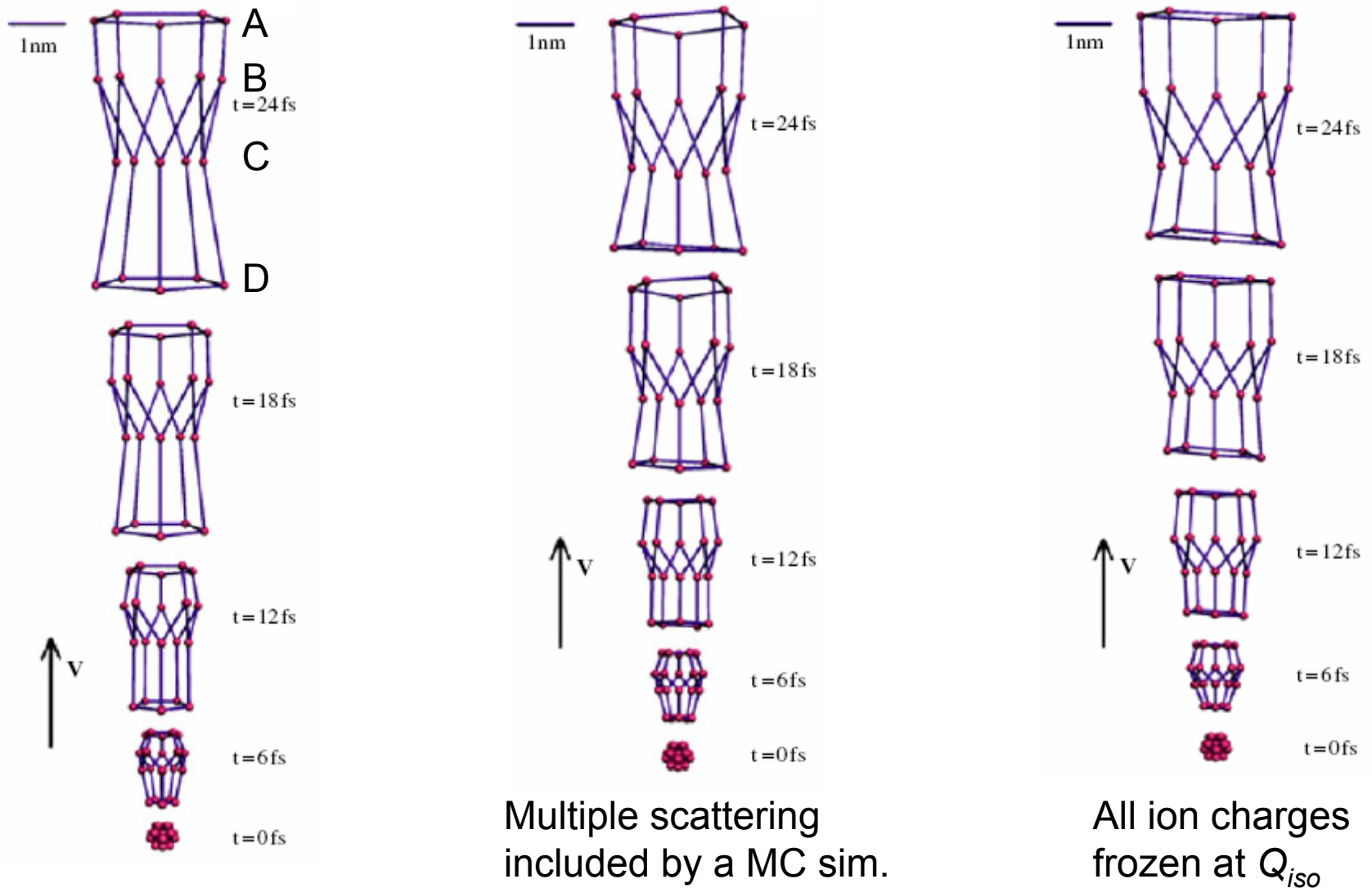


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# Charge states, Coulomb explosion, multiple scattering and energy loss of $C_{20}^+$ in Al foil

H.W. Li *et al.*, J. Phys.: Cond. Matt. 16 (2004) 1231

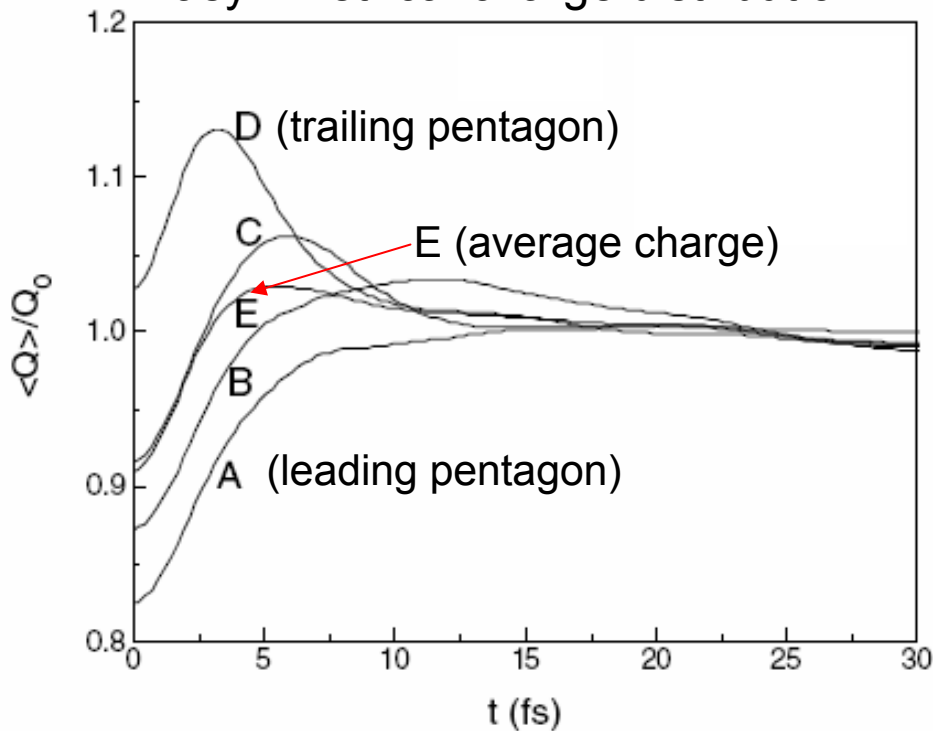




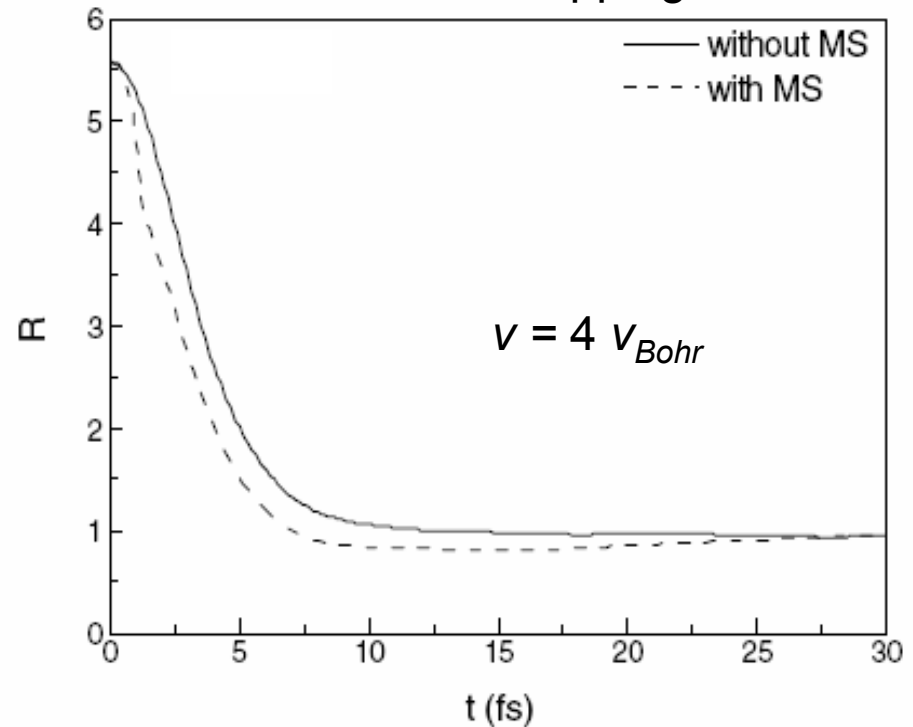
# Charge states, Coulomb explosion, multiple scattering and energy loss of $C_{20}^+$ in Al foil

H.W. Li *et al.*, J. Phys.: Cond. Matt. 16 (2004) 1231

Wake in inter-ionic potential gives asymmetrical charge distribution



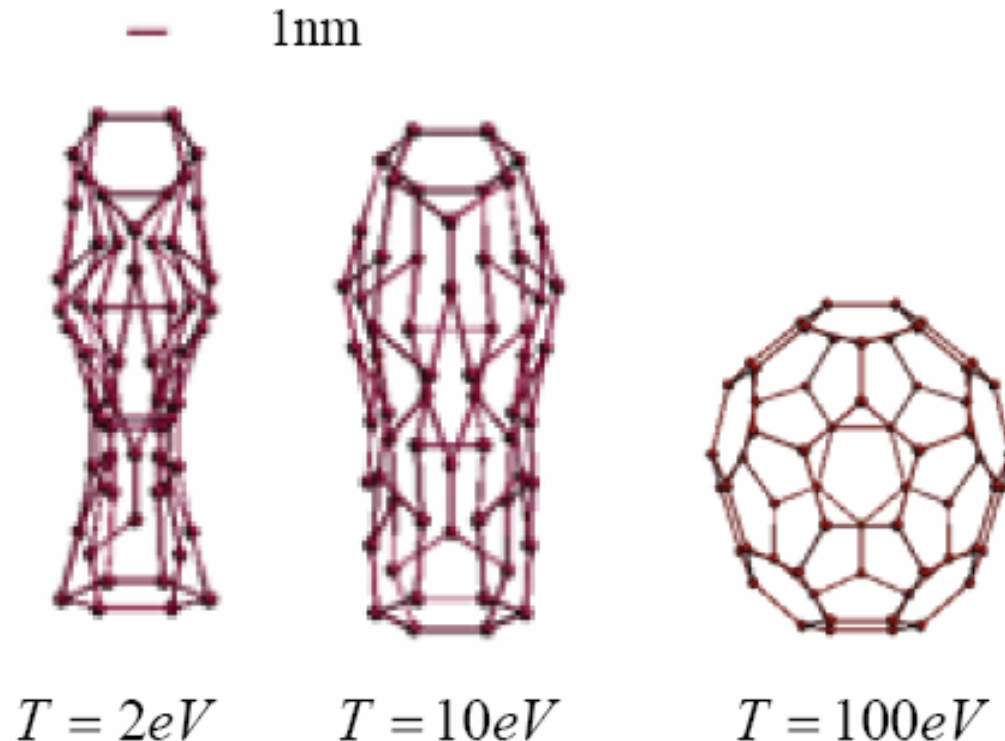
Effects of multiple scattering on reduction of stopping ratio



# Coulomb explosion of $C_{60}^+$ in hot dense plasma

G.Q. Wang *et al.*, Phys. of Plasmas 12 (2005) 42702

Effects of electron temperature after 25 fs



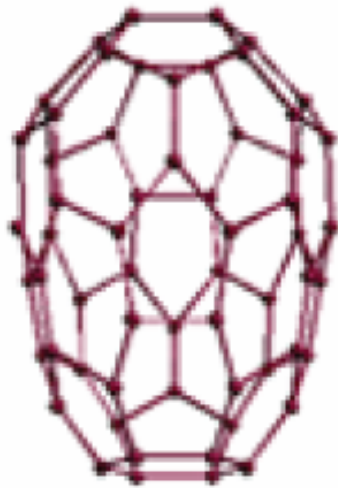
Other parameters: plasma density  $n = 10^{22} \text{ cm}^{-3}$ , cluster speed  $v = 3 v_T$

# Coulomb explosion of $C_{60}^+$ in hot dense plasma

G.Q. Wang *et al.*, Phys. of Plasmas 12 (2005) 42702

Effects of plasma density after 25 fs

— 1nm



$$n = 10^{21} \text{ cm}^{-3}$$



$$n = 5 \times 10^{21} \text{ cm}^{-3}$$



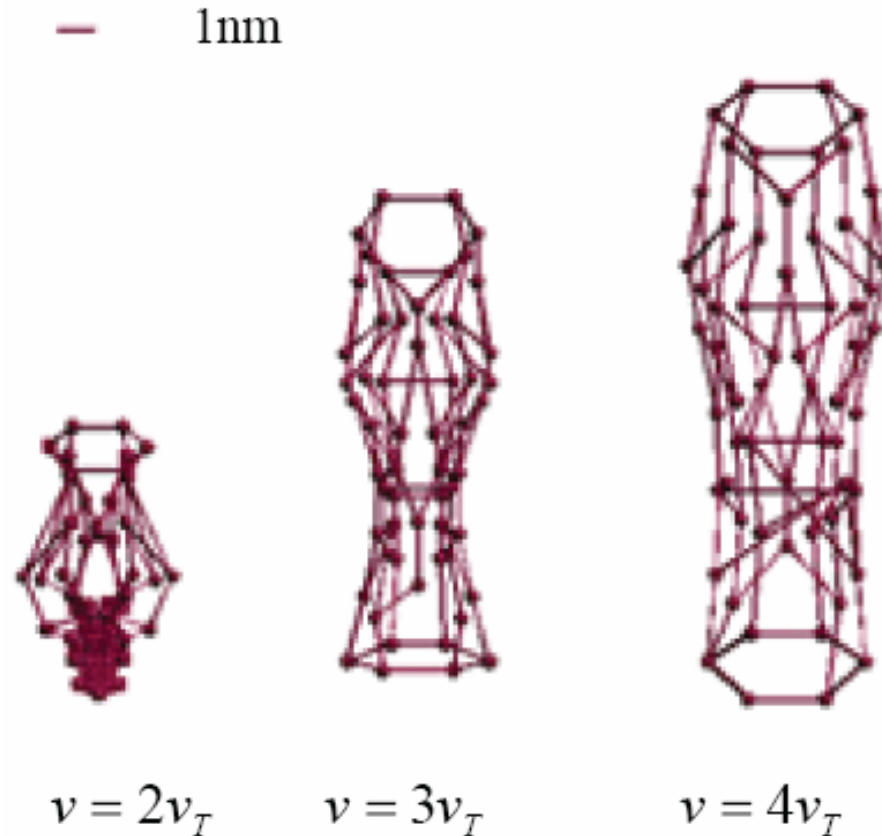
$$n = 10^{22} \text{ cm}^{-3}$$

Other parameters: electron temperature  $T = 2 \text{ eV}$ , cluster speed  $v = 3 v_T$

# Coulomb explosion of $C_{60}^+$ in hot dense plasma

G.Q. Wang *et al.*, Phys. of Plasmas 12 (2005) 42702

Effects of cluster speed after 25 fs



Other parameters: plasma density  $n = 10^{22} \text{ cm}^{-3}$ , electron temperature  $T = 2 \text{ eV}$

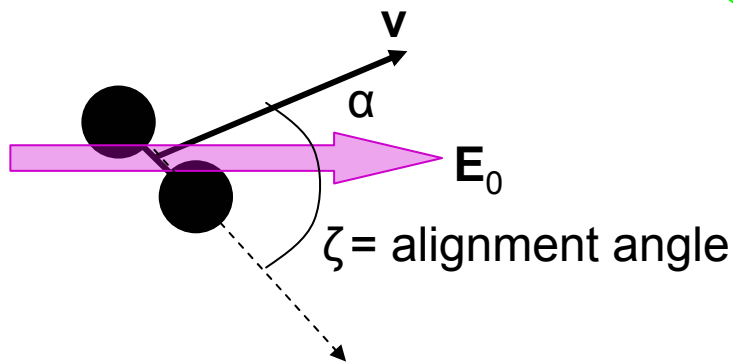
# Coulomb explosion of $H_2^+$ in laser-ablated Al plasma

experiments: A. Sakumi et al, NIMA 415 (1998) 648

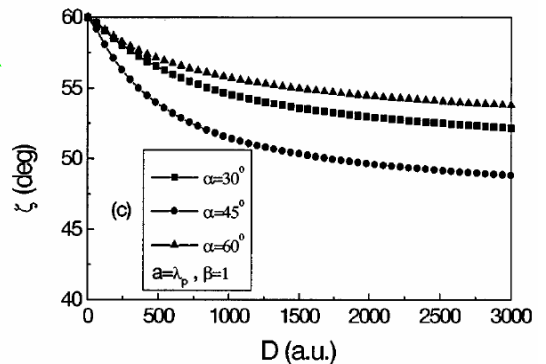
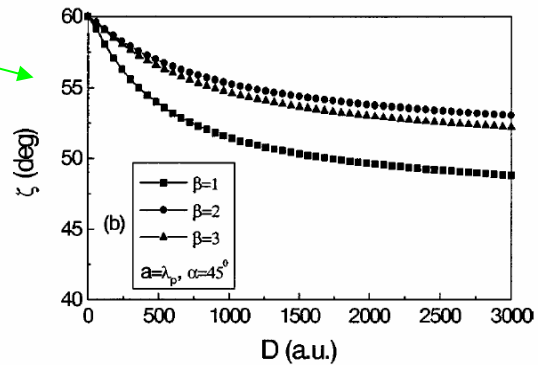
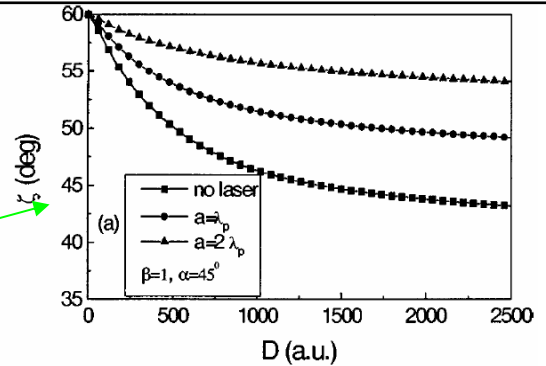
theory: G.Q. Wang *et al.*, PRA 66 (2002) 42901

## Effects of laser on molecule alignment

Effects:  $\left\{ \begin{array}{l} \text{laser intensity } a = E_0 / \omega_0^2 \\ \text{laser frequency } \beta = \omega_0 / \omega_p \\ \text{laser angle } \alpha \end{array} \right.$



$v = 3 \text{ a.u.}$   
 $\zeta_0 = 60^\circ$

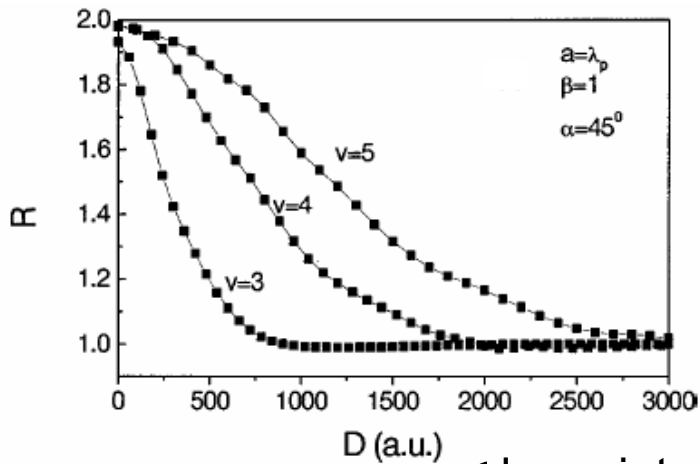


# Coulomb explosion of $H_2^+$ in laser-ablated Al plasma

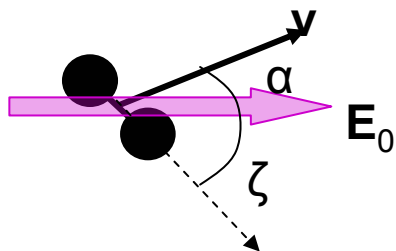
experiments: A. Sakumi *et al*, NIMA 415 (1998) 648

theory: G.Q. Wang *et al.*, PRA 66 (2002) 42901

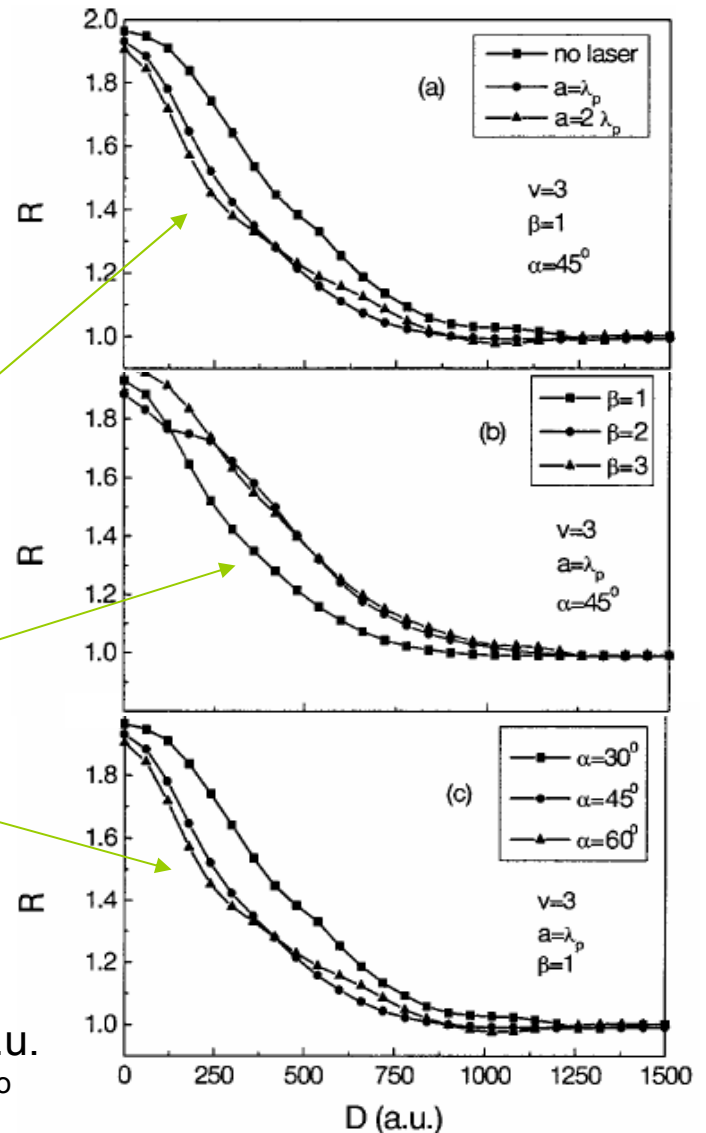
Effects of cluster speed (in a.u.) on **stopping ratio** vs penetration depth



Effects:  $\left\{ \begin{array}{l} \text{laser intensity } a = E_0 / \omega_0^2 \\ \text{laser frequency } \beta = \omega_0 / \omega_p \\ \text{laser angle } \alpha \end{array} \right.$



$v = 3$  a.u.  
 $\zeta_0 = 60^\circ$

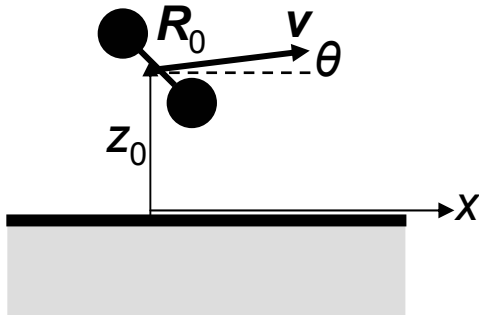


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# Grazing scattering of $N_2^+$ from carbon surface

Y.H. Song *et al.*, PRA 72 (2005) 12903



Trajectory of CM: 
$$\frac{dz_0}{dx_0} = \mp \theta \sqrt{1 - 2 \frac{U_{at}(z_0) + U_s(z_0) + U_w(z_0, \mathbf{R}_0)}{E\theta^2}}$$

Relative motion: 
$$\frac{d\mathbf{R}_0}{dx_0} = \frac{\mathbf{u}}{v}, \quad \frac{d\mathbf{u}}{dx_0} = \frac{1}{vm} (\mathbf{F}^{(e)} + \mathbf{F}^{(w)})$$

Molecule stopping power: 
$$S_{mol}(z_0, R_0) = 2S_e(z_0) + S_v(z_0, R_0)$$

Total energy loss  $\Delta E_{mol} = \int S_{mol}(z_0, R_0) ds$  gives stopping ratio  $\Gamma = \frac{\Delta E_{mol}}{2 \Delta E_{single}}$

Transition amplitude with step-potential wavefunctions and screened potential  $\Phi$

$$\mathbf{M}_{\mathbf{k} \leftarrow l_0} = \frac{-i}{(2\pi)^2} \frac{\kappa_z}{\kappa_z + l_z} \left[ 2 \left\langle \varphi_{l_z}^+ \mid \phi(\mathbf{Q}, \omega, z) \mid \varphi_{l_z^0}^0 \right\rangle + \frac{\kappa_z - l_z}{\sqrt{\kappa_z l_z}} \left\langle \varphi_{l_z}^- \mid \phi(\mathbf{Q}, \omega, z) \mid \varphi_{l_z^0}^0 \right\rangle \right]$$

Total electron emission probability: 
$$W_{mol} = \int \frac{ds}{v} \frac{d^3 P}{d\mathbf{k}^3}(x_0) = 2W_s + W_v$$

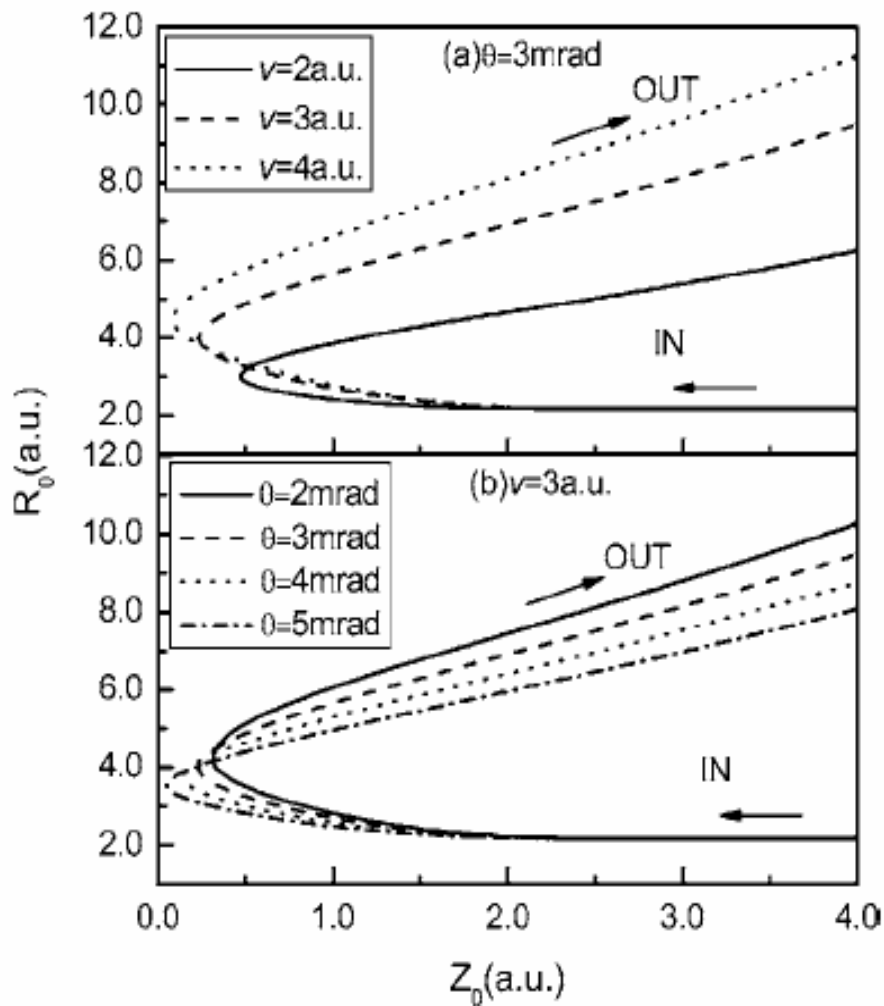
Charge state evolution: 
$$N(s) = N_\infty - (N_\infty - N_0) \exp[-n\sigma_c(s - s_0)]$$



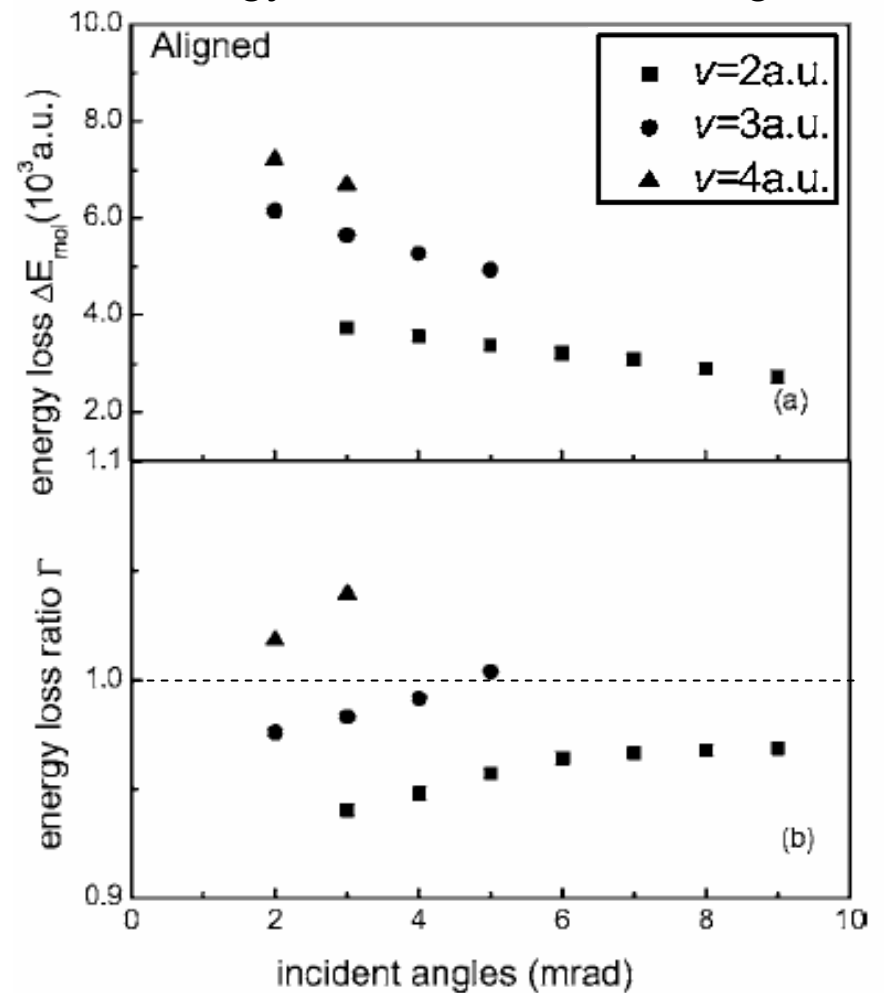
# Grazing scattering of $N_2^+$ from carbon surface

Y.H. Song *et al.*, PRA 72 (2005) 12903

Coulomb explosion of aligned molecules



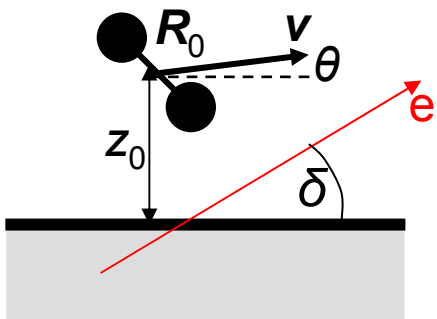
Energy losses vs incident angles



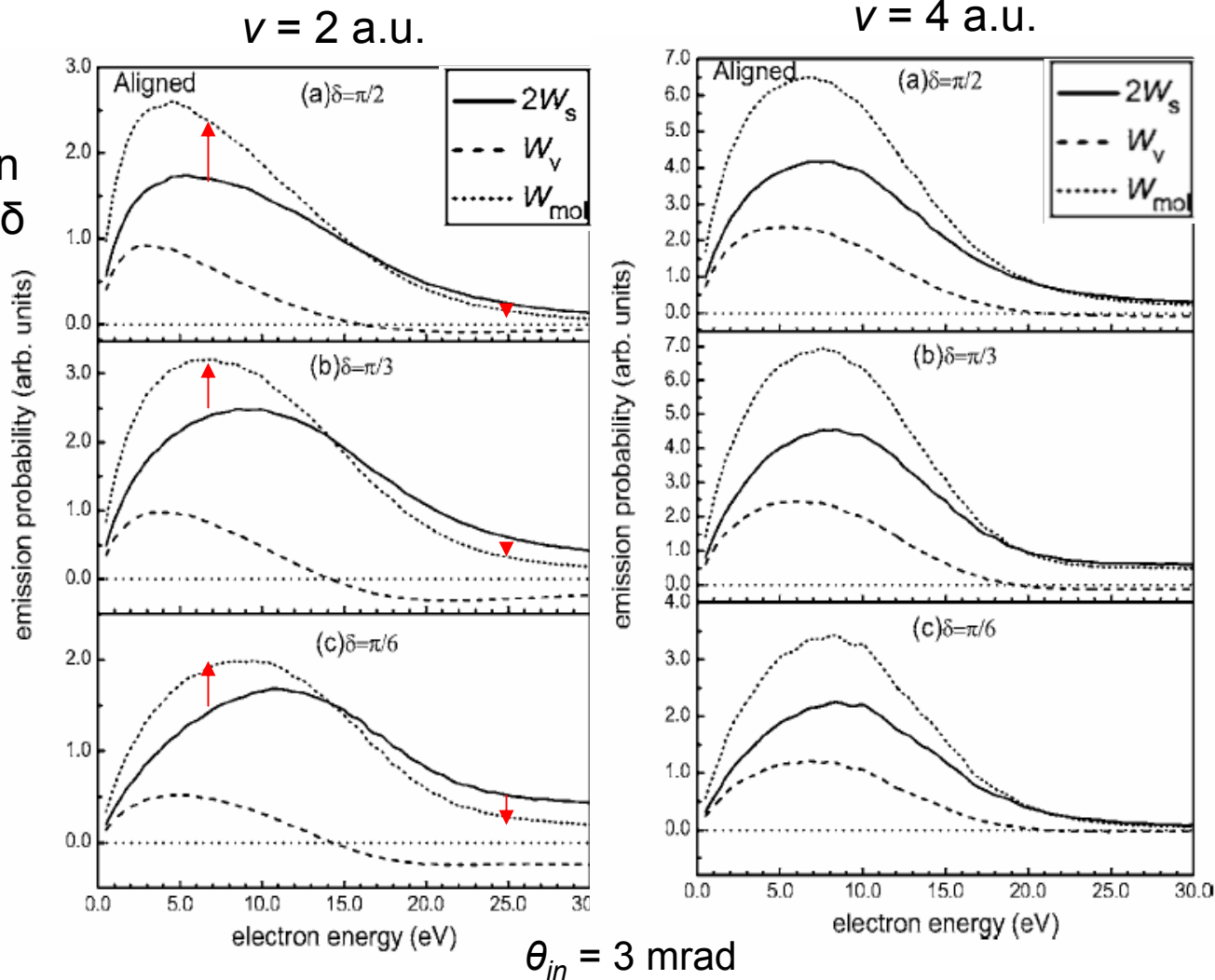
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Y.H. Song *et al.*, PRA 72 (2005) 12903

Vicinity effects on differential probability of Kinetic electron emission in the direction of angle  $\delta$



$$W_{mol} = 2W_s + W_v$$



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# Wake riding effect for dust particles in RF plasma sheath

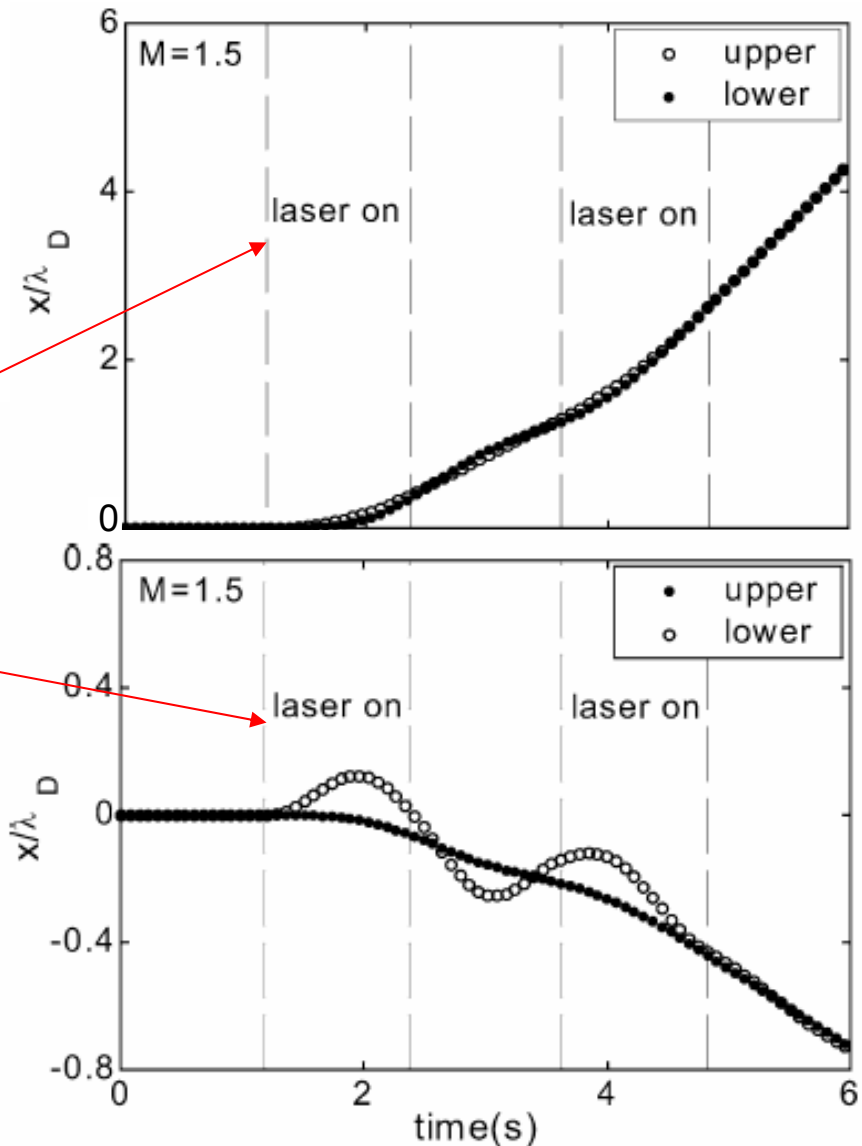
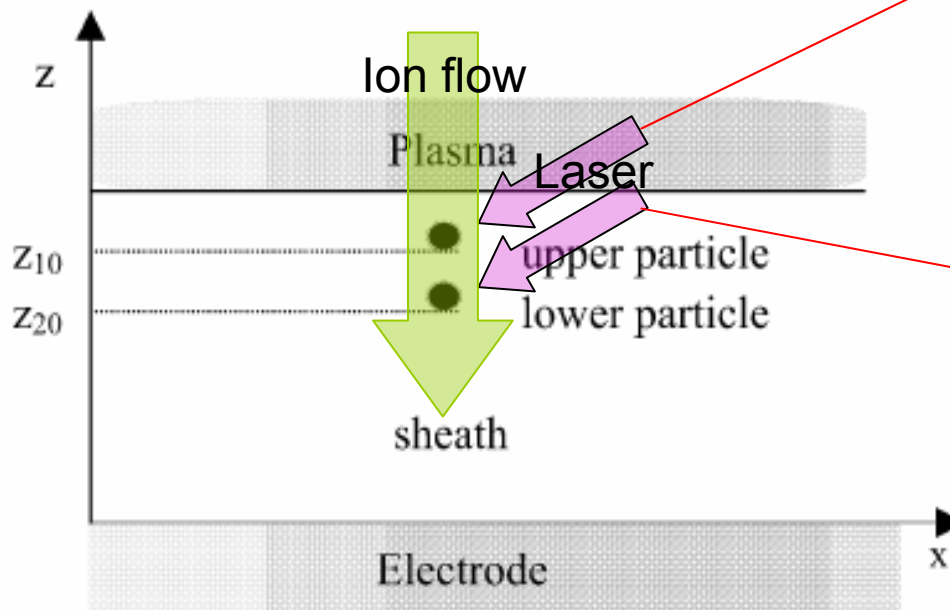
**experiment:** A.A. Samarian *et al.*, Phys. Plasmas, 12 (2005) 22103

**theory:** L.J. Hou *et al.*, PLA 292 (2001) 129

Find horizontal positions of particles from

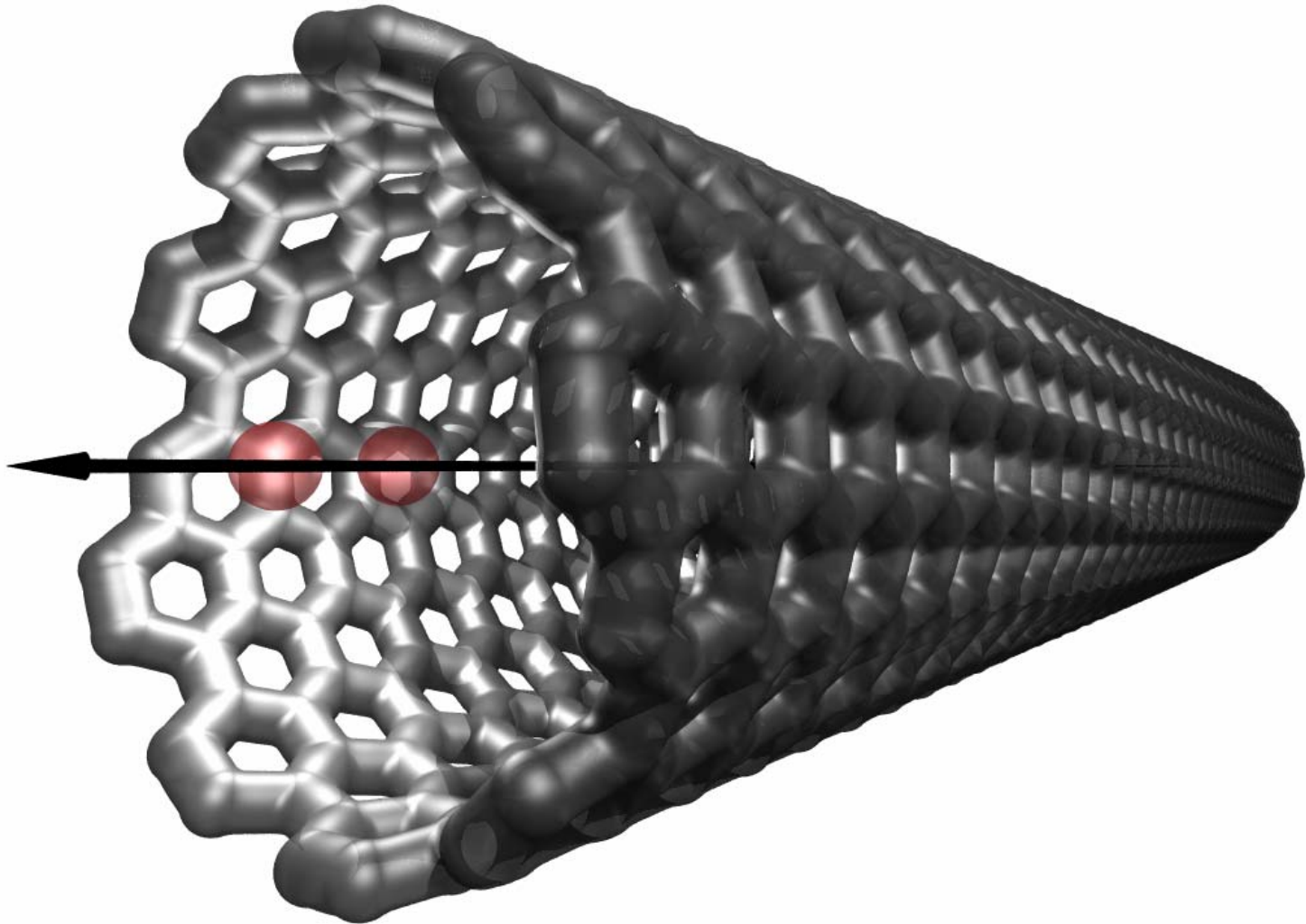
$$m_i \frac{d^2 x_i}{dt^2} = F_x(x_{ij}, z_{ij}) + F_n(u_{ix}) + F_{\text{opt}}$$

for  $z_{ij} = \text{const.}$ ,  $F_x$  exhibits asymmetry of wake between upper and lower particles



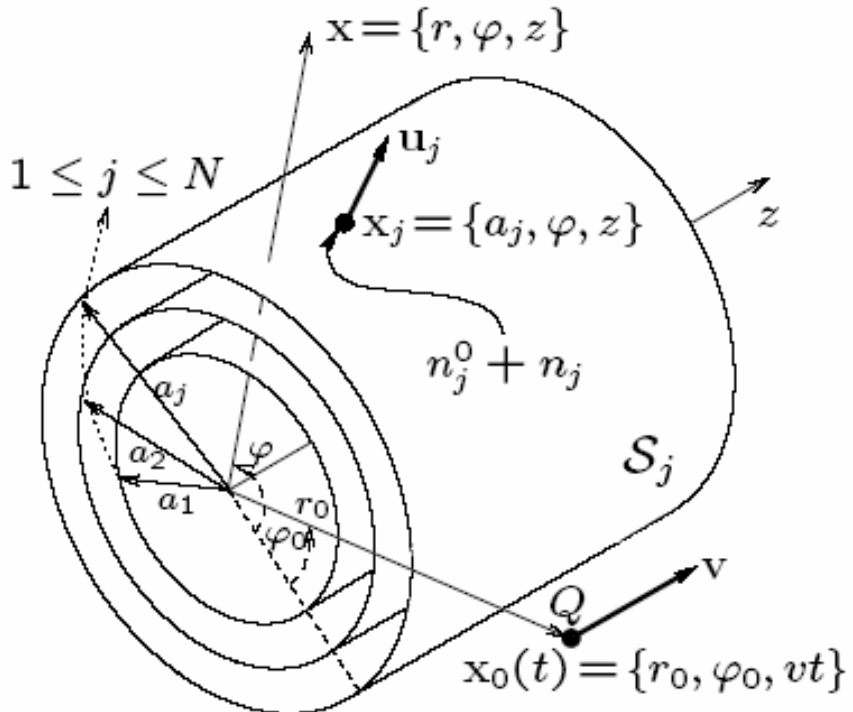
# Coulomb explosions during $H_2^+$ channelling in SWNT

D.P. Zhou *et al.*, PRA 73 (2006) 33202



# 2D hydrodynamic model of nanotube dielectric response

D.J. Mowbray *et al.*, PRB 70 (2004) 195418



$$\frac{\partial n_j(\mathbf{x}_j, t)}{\partial t} = -n_j^0 \nabla_j \cdot \mathbf{u}_j(\mathbf{x}_j, t)$$

$$\frac{\partial \mathbf{u}_j(\mathbf{x}_j, t)}{\partial t} = \nabla_j \Phi(\mathbf{x}, t)|_{r=a_j} - \frac{\alpha_j}{n_j^0} \nabla_j n_j(\mathbf{x}_j, t) + \frac{\beta}{n_j^0} \nabla_j [\nabla_j^2 n_j(\mathbf{x}_j, t)] - \gamma_j \mathbf{u}_j(\mathbf{x}_j, t)$$

$$\Phi(\mathbf{x}, t) = \frac{Q}{\|\mathbf{x} - \mathbf{x}_0(t)\|} - \sum_j \int_{S_j} d^2 \mathbf{x}'_j \frac{n_j(\mathbf{x}'_j, t)}{\|\mathbf{x} - \mathbf{x}'_j\|}$$

Stopping power

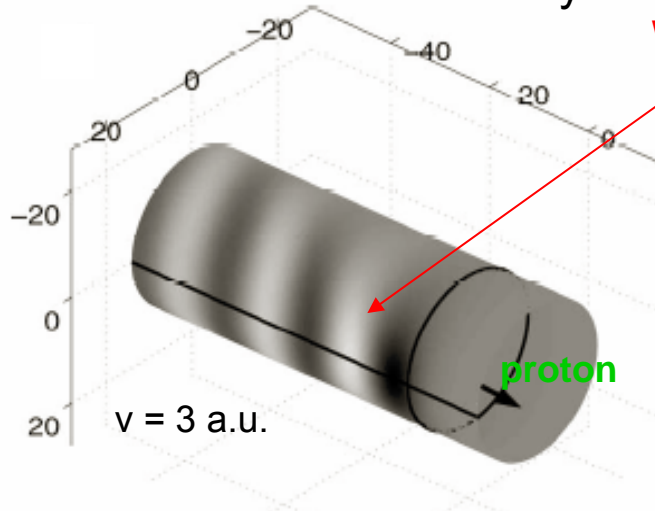
$$S = Q \frac{\partial \Phi_{ind}}{\partial z} \Big|_{\mathbf{x}=\mathbf{x}_0(t)}$$

Self-energy (image potential)

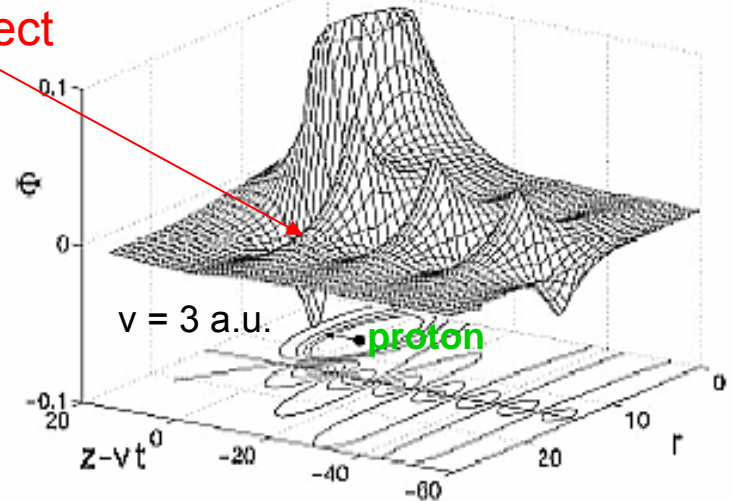
$$E_s = -\frac{Q}{2} \Phi_{ind} \Big|_{\mathbf{x}=\mathbf{x}_0(t)}$$

# Dynamic polarization of electrons on SWNT by proton

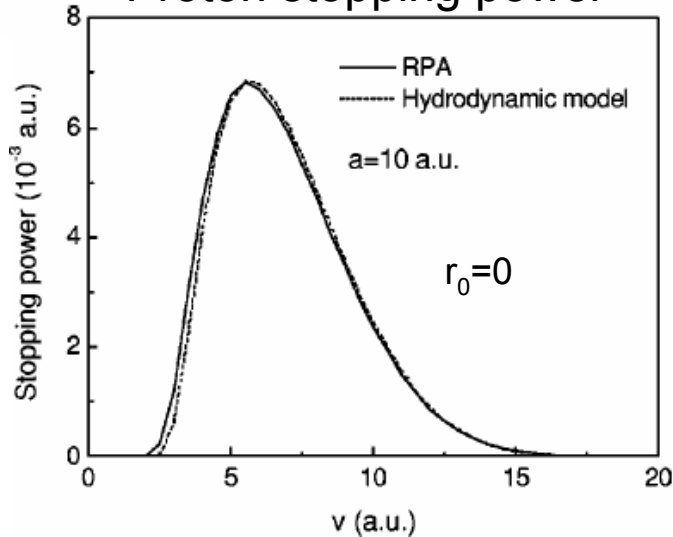
Induced electron density



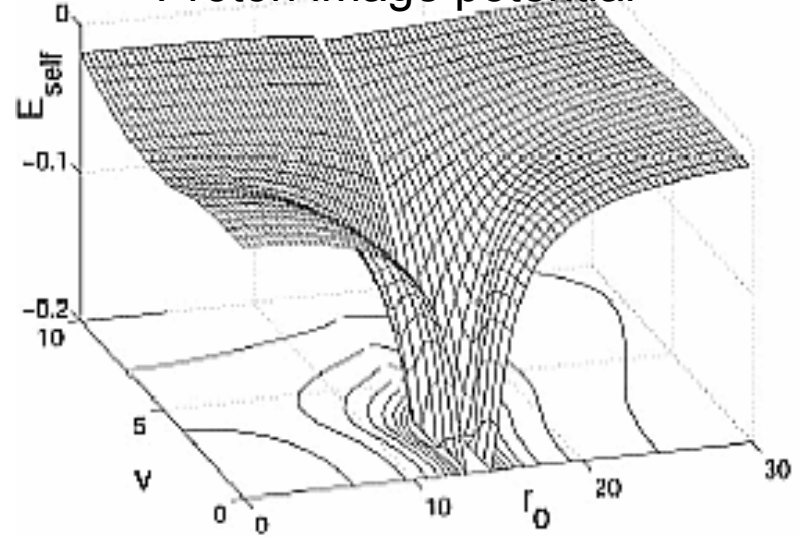
Total potential



Proton stopping power



Proton image potential



# Coulomb explosions during $H_2^+$ channelling in SWNT

D.P. Zhou *et al.*, PRA 73 (2006) 33202

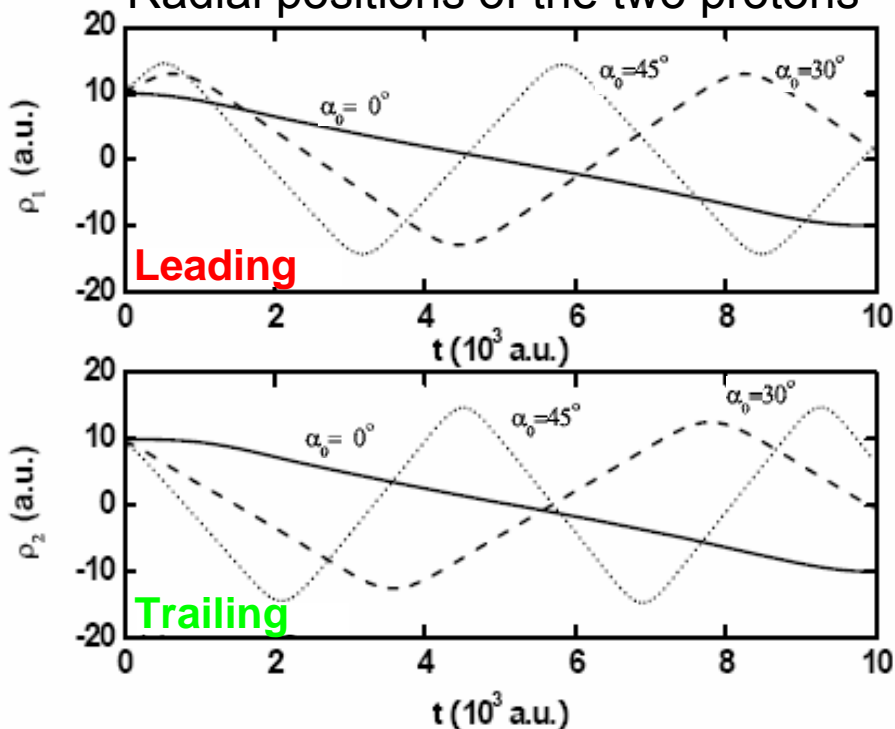
Solve classical equations of motion:

$$\frac{d\mathbf{r}_i}{dt} = \mathbf{u}_i, \quad M_i \frac{d\mathbf{u}_i}{dt} = \sum_{j(\neq i)=1}^2 \mathbf{F}_{ij}^{(c)} + \sum_{j=1}^2 \mathbf{F}_{ij}^{(p)} + \mathbf{F}_i^{(n)}$$

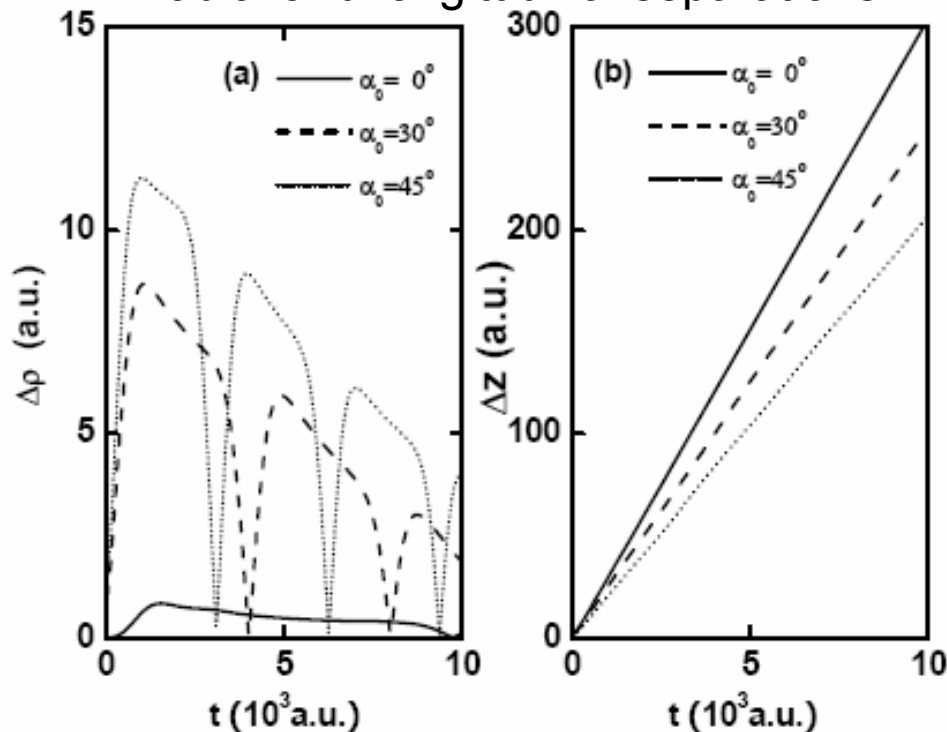
Molecule speed  $v_0 = 5$  a.u. and its alignment angles  $\alpha_0 = 0^\circ, 30^\circ, 45^\circ$

Forces: Coulomb, polarization, Moliere

Radial positions of the two protons



Radial and longitudinal separations

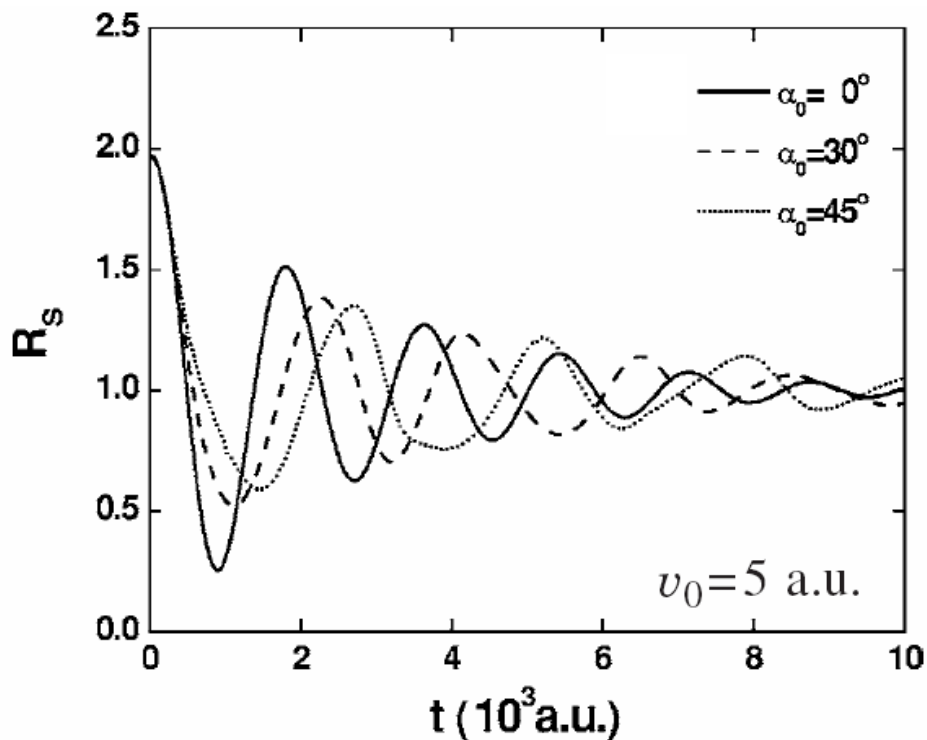




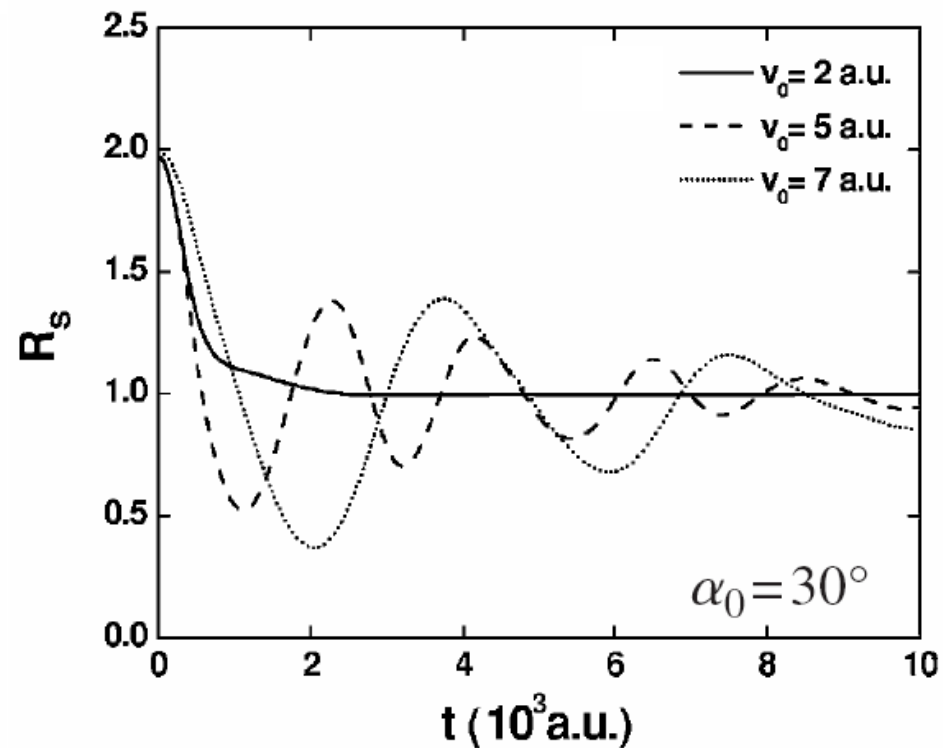
# Stopping power ratio vs dwell time for $H_2^+$ channelling in SWNT

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Effects of alignment angle  $\alpha_0$



Effects of molecule speed  $v_0$



# Outlook

- ❑ Vicinage effects:
  - Charge states (demonstrated in both exp. and theory)
  - Target polarization effect on Coulomb explosion patterns
  - Stopping power (exactly how strong? - still open question)
  
- ❑ Experiment: more systematic data on stopping needed
  
- ❑ Theory:
  - Charge-state modeling
    - ✓ transience effect
    - ✓ exit effect
    - ✓ fluctuations
  - Dielectric response models:
    - ✓ inner-shell corrections
    - ✓ non-linearity ( $Z_1^3$  effect)
    - ✓ new targets
  - Going beyond dielectric models